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TECHNICAL
REPORT

COLORIMETRY

CIE 15:2004
3rd Edition

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This Technical Report has been prepared by CIE Technical Committee 1-48 of Division 1 "Vision and Colour" and has been approved by the Board of Administration of the Commission Internationale de l'Eclairage for study and application. The document reports on current knowledge and experience within the specific field of light and lighting described, and is intended to be used by the CIE membership and other interested parties. It should be noted, however, that the status of this document is advisory and not mandatory. The latest CIE proceedings or CIE NEWS should be consulted regarding possible subsequent amendments.

Ce rapport technique a été élaboré par le Comité Technique CIE 1-48 de la Division 1 "Vision et Couleur" et a été approuvé par le Bureau de la Commission Internationale de l'Eclairage, pour étude et emploi. Le document expose les connaissances et l'expérience actuelles dans le domaine particulier de la lumière et de l'éclairage décrit ici. Il est destiné à être utilisé par les membres de la CIE et par tout les intéressés. Il faut cependant noter que ce document est indicatif et non obligatoire. Il faut consulter les plus récents comptes rendus de la CIE, ou le CIE NEWS, en ce qui concerne des amendements nouveaux éventuels.

Dieser Technische Bericht ist vom CIE Technischen Komitee 1-48 der Division 1 "Sehen und Farbe" ausgearbeitet und vom Vorstand der Commission Internationale de l'Eclairage gebilligt worden. Das Dokument berichtet über den derzeitigen Stand des Wissens und Erfahrung in dem behandelten Gebiet von Licht und Beleuchtung; es ist zur Verwendung durch CIE-Mitglieder und durch andere Interessierte bestimmt. Es sollte jedoch beachtet werden, daß das Dokument eine Empfehlung und keine Vorschrift ist. Die neuesten CIE-Tagungsberichte oder das CIE NEWS sollten im Hinblick auf mögliche spätere Änderungen zu Rate gezogen werden.

This report has been prepared by the Technical Committee 1-48 "Revision of CIE document 15.2 Colorimetry" of CIE Division 1 "Colour and Vision" by using previously published material of the CIE and considering resolutions of the CIE Division 1 meeting at Teddington 2000-04-06 and Rochester 2001-06-22/23. This present publication replaces CIE 15.2-1986 "Colorimetry"[1].

Members of the Technical Committee during the preparation of this report were:

P.J. Alessi	USA
E.C. Carter	USA
M.D. Fairchild	USA
R.W.G. Hunt	United Kingdom
C.S. McCamy	USA
B. Kránicz	Hungary
J.R. Moore	United Kingdom
L. Morren	Belgium
J.H. Nobbs	United Kingdom
Y. Ohno	USA
M.R. Pointer	United Kingdom
D.C. Rich	USA
A.R. Robertson	Canada
J.D. Schanda (chair)	Hungary
R. Sève	France
P.W. Trezona	United Kingdom
K. Witt	Germany
H. Yaguchi	Japan

The following Editorial Committee was responsible for the formulation of the document: E.C. Carter, Y. Ohno, M.R. Pointer, A.R. Robertson, R. Sève, J.D. Schanda, K. Witt.

Items of mainly historic importance have been placed into Appendix A.

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[1] According to the new CIE publication numbering policy - that indicates a new revised edition only by the year of publication - this technical report has got the number of 15:2004 and not 15.3:2004.

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COLORIMETRY

SUMMARY

This publication provides the recommendations of the CIE concerning basic colorimetry. Specifically, it includes the use of the standard illuminants and the standard colorimetric observers; the reference standard for reflectance; the illuminating and viewing conditions; the calculation of tristimulus values, chromaticity coordinates, colour spaces and colour differences; and the various other colorimetric practices and formulae.

This publication is consistent with the fundamental data and procedures described in the CIE Standards on Colorimetry.

For further details of some of the phenomena discussed here the reader is directed to the appropriate technical reports.

COLORIMETRIE

RESUME

Ce document donne les recommandations de la CIE pour la colorimétrie de base. Plus particulièrement il traite de l'emploi des illuminants normalisés et des observateurs colorimétriques normalisés, de la référence normalisée pour le facteur de réflexion, des conditions d'éclairage et d'observation, du calcul des composantes trichromatiques, des coordonnées trichromatiques, des espaces chromatiques et des écarts de couleur, ainsi que de l'emploi de diverses autres pratiques et formules colorimétriques.

Cette publication est en accord avec les données et procédures de base qui sont décrites dans les normes CIE de colorimétrie.

Pour des détails complémentaires relatifs aux questions traitées ici, le lecteur est invité à consulter les rapports techniques appropriés.

FARBMESSUNG

ZUSAMMENFASSUNG

Die hier wiedergegebenen Empfehlungen der CIE beziehen sich auf die Anwendung oder die Benutzung der Normlichtarten und der Normalbeobachter, auf den Weißstandard, auf die Beleuchtungs- und Beobachtungsbedingungen, auf die Berechnung von Normfarbwerten und Normfarbwertanteilen, Farbenräumen und Farbabständen, und auf verschiedene andere farbmetrische Praktiken und Formeln.

Diese Veröffentlichung stimmt mit den grundlegenden Daten und Verfahren überein, die in CIE Normen für Farbmessung beschrieben sind.

Weitere Einzelheiten einiger hier diskutierter Phänomene findet der Leser in geeigneten technischen Berichten.

1. SCOPE

This report is intended to provide a consistent and comprehensive account of the recommendations of the CIE for basic colorimetry. It summarises basic colorimetric data and practices; it does not, however, deal with colour appearance specification.

2. PREFACE

By general consent in all countries the specification of basic standards for use in colorimetry is the province of the Commission Internationale de l'Eclairage (CIE). The first major recommendations regarding colorimetric standards were made by the CIE in 1931, and these formed the basis of modern colorimetry. The original recommendations of 1931 were reviewed from time to time by the CIE Colorimetry Committee and later by CIE Division 1, *Vision and Colour*. When necessary changes were made. New recommendations were added to supplement the existing ones or to broaden the scope of colorimetry in accordance with developments in practice and science.

The deliberations and recommendations made by the CIE Colorimetry Committee and Division 1 are recorded in the Proceedings of the various Sessions of the CIE. However the access to these Proceedings has always been rather limited and much of the material published in the Proceedings is obsolete or inconsistent with current colorimetric practice. The recommendations, while few in number, also present an incoherent picture. Many recommendations are merely proposals of study or work on certain topics that were considered important at the time.

For these reasons in 1971 the CIE published a special document on colorimetry to provide a consistent and comprehensive account of basic colorimetric recommendations. This document was not intended to be a textbook on colorimetry but rather a reference to the basic standards that govern modern colorimetry. The document was issued as CIE Publication No. 15 (CIE, 1971).

Since 1971 it has been necessary to add two supplements to the document [Suppl. 1 on *Special metamerism index: change of illuminant* (CIE, 1972) and Suppl. 2 on *Recommendations on uniform colour spaces, colour-difference equations and psychometric colour terms* (CIE, 1978)]. Several Technical Reports were also published during the intervening years. Publication CIE 15.2, published in 1986 (CIE, 1986b) incorporated the first two supplements. Further amendments, described in detail in CIE 51-1981, *A method for assessing the quality of daylight simulators for colorimetry* (CIE, 1981); CIE 80-1989, *Special metamerism index: observer metamerism* (CIE, 1989); CIE 101-1993, *Parametric effects in colour difference evaluation* (CIE, 1993); CIE 116-1995, *Industrial colour difference evaluation* (CIE, 1995a); CIE 135/3, *CIE TC 1-45 report: Supplement 1-1999 to CIE 51-1981, Virtual metamers for assessing the quality of simulators of CIE illuminant D50* (CIE, 1999b) and CIE 142-2001, *Improvement to industrial colour difference evaluation* (CIE, 2001a) are all parts of the CIE system of colorimetry. The present version, CIE 15:2004 summarises all CIE recommendations on basic colorimetry. Short explanatory texts and a historic overview are also included. Colour appearance models are not covered, for these see CIE 131-1998 (CIE, 1998a); CIE 159:2004 (CIE, 2004b) and CIE x014-1998 (CIE, 1998b).

In compiling this third edition the opportunity has been taken to incorporate all relevant recommendations. The publications enumerated in the previous paragraph contain background information on the single subjects, those interested in more detail about the development of the recommendations and their background experiments are referred to these publications. CIE 15:2004 is based on CIE standards containing the fundamental data on CIE standard illuminants and standard observers. The presently official versions of these standards are CIE S 005-1998, *CIE standard illuminants for colorimetry* (CIE, 1998c) (published also as ISO 10526:1999) and CIE S 002-1986, *CIE standard colorimetric observers* (CIE, 1986a), (published also as CIE/ISO 10527:1991). These standards contain the fundamental colorimetric data [see also CIE DS 014-2.2:2004 (CIE, 2004a)].

It is anticipated that further amendments of colorimetric practice and further standards will be published, thus one should consult the latest CIE list of publications published in CIE Proceedings, and in most recent Technical Reports; this information is also available on the CIE world wide web home page (http://www.cie.co.at/).

The wording of the original recommendations has been altered to be consistent with modern nomenclature and in some cases the original recommendations have also been modified in content to bring them into line with present day thinking and practice. The versions given in this document are the recommendations now in force and supersede all previous recommendations until such time as any amendments are published by the CIE.

The recommendations are divided into the following seven groups:

- Recommendations concerning standard physical data of illuminants and sources.
- Recommendations concerning the standard of reflectance.
- Recommendations concerning geometric conditions for colorimetry.
- Recommendations concerning standard observer data.
- Recommendations concerning the calculation of tristimulus values and chromaticity coordinates.
- Recommendations concerning uniform colour spacing and colour difference.
- Recommendations concerning miscellaneous colorimetric practices and formulae.

The definitive data relating to standard colorimetric illuminants and observers are those given at 1 nm intervals in the appropriate CIE standards [CIE, 1998c (to be replaced by CIE, 2004a); CIE, 1986a]. The data have not been printed in the present report, but are included in the accompanying CD-ROM. Abridged data, at 5 nm intervals, are given in Section 11, and these may be used in those cases where calculation at the wider intervals is unlikely to produce any significant error.

For all colorimetric calculations wavelength in standard air should be used[II] [1].

3. RECOMMENDATIONS CONCERNING STANDARD PHYSICAL DATA OF ILLUMINANTS AND SOURCES

3.1 Recommendations concerning standard physical data of illuminants[2]

It is recommended that the following illuminants, defined by relative spectral power distributions given in Table 1 of CIE S 005-1998 (CIE, 1998c)[III], be used for general colorimetry.

- **CIE standard illuminant A**

The relative spectral power distribution $S_A(\lambda)$ is defined by the equation

$$S_A(\lambda) = 100\left(\frac{560}{\lambda}\right)^5 \times \frac{\exp\frac{1{,}435\times10^7}{2\,848\times560}-1}{\exp\frac{1{,}435\times10^7}{2\,848\,\lambda}-1} \qquad (3.1)$$

where λ is the wavelength in nanometres and the numerical values in the two exponential terms are definitive constants originating from the first definition of Illuminant A in 1931[3]. This spectral power distribution is normalized to the value 100 (exactly) at the wavelength 560 nm (exactly).

CIE standard illuminant A is defined over the spectral region from 300 nm to 830 nm[IV] in CIE S 005-1998 (CIE, 1998c) to six significant digits. Should higher precision be needed,

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[II] Superscripts of Arabic numerals refer to Explanatory Comments given on page 68-72.

[III] The spectral power distribution tables published in the CIE Standard are reproduced on the CD-ROM that accompanies this Technical Report.

[IV] The wavelength range for standard illuminants is 300 nm – 830 nm because, for evaluating luminescent samples the UV spectral range is relevant. For most colorimetric investigations the restricted wavelength range, 380 nm – 780 nm, can be used.

values calculated using Equ. 3.1 should be used[V]. Section 11 contains abridged tables that can be used in most practical calculations. On the use of these tables see Section 7.2.

Note 1: The definition reproduced here is in accordance with the values of Illuminant A originally published in 1931, see CIE S 005-1998 (CIE, 1998c) and CIE DS 014-2.2:2004 (CIE, 2004a).

Note 2: Despite the fact that Equ. 3.1 is based on Planck's equation for a vacuum, the wavelengths are to be taken as being in standard air (dry air at 15°C and 101325 Pa, containing 0,03% by volume of carbon dioxide). This makes CIE standard illuminant A compatible with other CIE colorimetric and photometric data.

- **CIE standard illuminant D65**

The relative spectral power distribution representing a phase of daylight with a correlated colour temperature of approximately 6500 K (called also nominal correlated colour temperature of the daylight illuminant)[4], symbol: $S_{D65}(\lambda)$. The official values of $S_{D65}(\lambda)$ are as given in CIE standard illuminants for colorimetry (CIE, 1998c)[VI].

Note 1: Regarding the definition of correlated colour temperature (T_{cp}) of an illuminant see Section 9.5.

Note 2: Section 11 provides abridged tables that can be used in many practical calculations. On the use of these tables see Section 7.2. The values of relative spectral power distribution of CIE standard illuminant D65 given in Table T.1 at 5 nm intervals are consistent with the values from 300 nm to 830 nm at 1 nm intervals and with six significant figures given in CIE standard illuminants for colorimetry (CIE, 1998c). They have been taken from the tables of the Standard.

Note 3: If values at other wavelengths than printed in Table 1 of the standard (CIE, 1998c) at 1 nm intervals are needed, linear interpolation should be used.

- **Other illuminants D**

It is recommended that, in the interest of standardization, D65 be used whenever possible. When D65 cannot be used, it is recommended that one of the daylight illuminants D50, D55, or D75 defined in Table T.1 be used. When none of these daylight illuminants can be used, a daylight illuminant at a nominal correlated colour temperature (T_{cp}) can be calculated using the following equations. These equations will give an illuminant whose correlated colour temperature is approximately equal to the nominal value, but not exactly so[4].

(a) Chromaticity

The 1931 (x,y) chromaticity coordinates of the daylight (D) to be defined must satisfy the following relation:

$$y_D = -3{,}000 x_D^2 + 2{,}870 x_D - 0{,}275 \qquad (3.2)$$

with x_D being within the range of 0,250 to 0,380. The correlated colour temperature T_{cp} of daylight D is related to x_D by the following formulae based on normals to the Planckian locus on a uniform chromaticity diagram (see Section 9.5):

(i) for correlated colour temperatures from approximately 4000 K to 7000 K:

$$x_D = \frac{-4{,}6070 \times 10^9}{(T_{cp})^3} + \frac{2{,}9678 \times 10^6}{(T_{cp})^2} + \frac{0{,}09911 \times 10^3}{(T_{cp})} + 0{,}244063 \qquad (3.3)$$

(ii) for correlated colour temperatures from greater than 7000 K to approximately 25 000 K:

[V] Terminology in this publication follows the traditional terminology used in colorimetry. A separate publication on colorimetry will deal with the current "uncertainty" description of stating measurement results and will recommend methods for calculating colorimetric uncertainties.

[VI] The spectral power distribution tables published in the CIE Standard are reproduced on the CD-ROM that accompanies this Technical Report.

$$x_D = \frac{-2{,}0064 \times 10^9}{(T_{cp})^3} + \frac{1{,}9018 \times 10^6}{(T_{cp})^2} + \frac{0{,}24748 \times 10^3}{(T_{cp})} + 0{,}237040 \qquad (3.4)$$

(b) Relative spectral power distribution

The relative spectral power distribution $S(\lambda)$ of daylight D is to be computed from

$$S(\lambda) = S_0(\lambda) + M_1 S_1(\lambda) + M_2 S_2(\lambda) \qquad (3.5)$$

where $S_0(\lambda)$, $S_1(\lambda)$, $S_2(\lambda)$ are functions of wavelength, λ, given in Table T.2, and M_1, M_2 are factors whose values are related to the chromaticity coordinates x_D, y_D as follows:

$$M_1 = \frac{-1{,}3515 - 1{,}7703 x_D + 5{,}9114 y_D}{0{,}0241 + 0{,}2562 x_D - 0{,}7341 y_D}$$

$$M_2 = \frac{0{,}0300 - 31{,}4424 x_D + 30{,}0717 y_D}{0{,}0241 + 0{,}2562 x_D - 0{,}7341 y_D} \qquad (3.6)$$

Notes on standard illuminant D65 and other illuminants D

Note 1: Seasonal and geographical variations in the spectral power distribution of daylight occur, particularly in the ultraviolet spectral region, but this recommendation should be used pending the availability of further information on these variations.

Note 2: The spectral power distributions of daylight illuminants D produced by this recommendation are based on experimental observations over the wavelength range 330 nm to 700 nm, and on extrapolation in the wavelength ranges 300 nm to 330 nm and 700 nm to 830 nm (see Judd et al., 1964). The extrapolated values are believed to be accurate enough for colorimetric purposes, but should not be used for other purposes.

Note 3: If values at other wavelengths than those printed in Table T.2 are needed, linear interpolation should be used (see also Appendix C).

Note 4: The relative spectral power distributions of the D illuminants given in Table T.1 and in the CIE standard on illuminants for colorimetry (CIE, 1998c) were derived by the procedure given above with some intermediate rounding and with some adjustments for changes in the International Temperature Scale[5]. Thus for historic reasons, the tabulated values are slightly different from the calculated values. For the time being the tabulated values are the official data. (Eventually these data may be superseded by the method described in Appendix C, which provides a harmonised method of calculation for trial that will produce congruent results if the modified tables and equations are used.)

Note 5: When samples exhibiting luminescence excited by ultraviolet radiation are involved, one of the D illuminants defined in these recommendations should always be used to represent daylight.

- **Illuminant B**

Intended to represent direct sunlight with a correlated colour temperature of approximately 4900 K.

Note: The use of this illuminant is deprecated, see Appendix A.

- **Illuminant C**

Intended to represent average daylight with a correlated colour temperature of approximately 6800 K.

Note 1: Illuminant C does not have the status of a CIE standard but its relative spectral power distribution, tristimulus values and chromaticity coordinates are given in Table T.1 and Table T.3, as many practical measurement instruments and calculations still use this illuminant.

Note 2: The tristimulus values and chromaticity coordinates of the illuminant C and of D50, D55, D75, calculated from the values of $S(\lambda)$, given in Table T.2., are shown in Table T.3. The reproduced tristimulus values are those of the perfect reflecting diffuser irradiated by the respective illuminant, see Section 7.1.1. This table also includes similar data for CIE standard illuminant A and D65.

3.2 Artificial sources representative of illuminants

It is recommended that the following artificial sources be used if it is desired to realise the illuminants defined in Section 3.1 for actual laboratory inspection.

- **Source A**

CIE standard illuminant A is to be realised by a gas-filled tungsten filament lamp operating at a correlated colour temperature of 2856 K ($c_2 = 1{,}4388 \times 10^{-2}$ m·K). If the source is also to be used in the UV region, a lamp having an envelope or window made of fused-quartz or silica must be used because glass absorbs the UV component of the radiation from the filament.

- **Source B**

See Appendix A.

- **Source C**

See Appendix A.

- **Source D65**

At present no artificial source is recommended to realise CIE standard illuminant D65 or any other illuminant D of different correlated colour temperature. It is hoped that new developments in light sources and filters will eventually offer sufficient basis for a CIE recommendation. Meanwhile the CIE has agreed on a formula to describe the quality of a daylight simulator for colorimetry, see Section 9.3.

Notes on artificial sources representative of illuminants

Note 1: The artificial sources defined above and recommended as representative sources for CIE illuminants are named "CIE sources for colorimetry".

Note 2: Whenever the highest accuracy of the spectral power distribution of a standard is required, it is advisable to make a spectroradiometric calibration of the actual source used, because the relative spectral power distribution of the source may not exactly coincide at all wavelengths with that defining the corresponding illuminant.

Note 3: The spectrum of a D65 simulator fluorescent lamp is reproduced in Table T.6.2b as FL3.15, see Section 9.3.

4. RECOMMENDATIONS CONCERNING STANDARD OF REFLECTANCE[6]

The perfect reflecting diffuser is the reference standard for reflectance (CIE, 1986a). It is defined as the ideal isotropic diffuser with a reflectance equal to unity. For real measurements, reflectance standards, such as pressed barium sulphate or PTFE (known also under the trade names Algoflon, Halon, Spectralon), must be calibrated in terms of the perfect reflecting diffuser (see CIE, 1979a; CIE, 1979b) for the required geometry.

5. RECOMMENDATIONS CONCERNING GEOMETRIC CONDITIONS FOR COLORIMETRY[7]

Colorimetric specifications are derived from spectral or tristimulus measurements. The measured values depend on the geometric relationships between the measuring instrument and the sample. These relationships are called "geometric conditions" or simply "geometry". Similarly, visual appraisals of coloured samples are affected by illuminating and viewing geometry. The degree of correlation between measured values and visual appraisals depends on the degree to which the geometric conditions of measurement simulate the geometric conditions of viewing. (The use of the term "viewing" with reference to measuring instruments is deprecated because it blurs the important distinction between instrumental measurements and visual observation.)

Terms and definitions used in this Section of CIE 15:2004 introduce a terminology not used in previous versions of CIE 15, they are the following:

reference plane

> plane in which the surface of a sample or standard is placed during measurements
> For reflection measurements, the geometry is defined with respect to the reference plane. For transmission measurements, there is a reference plane for the incident light and a second reference plane, displaced by the sample thickness, for the transmitted light. The recommendations of this publication are based on the assumption of negligible thickness.

sampling aperture

> area of the reference plane on which measurements are made
> The sampling aperture is delimited by the area illuminated or by the area over which the receiver senses flux, whichever is smaller. If the illuminated area is the larger, the area measured is said to be "over filled"; if it is the smaller, the area measured is said to be "under filled".

modulation

> generic term for measured ratios such as reflectance, reflectance factor or transmittance

irradiation or influx (illumination or incidence) geometry

> angular distribution of irradiance at the centre of the sampling aperture

reflection/transmission or efflux (collection, measuring) geometry

> angular distribution of receiver responsivity with respect to the centre of the sampling aperture

Note: The influx and efflux geometry specify the geometric nature of the measurement.

It is recommended that the geometry used when determining the colorimetric specification of diffusely reflecting samples corresponds to one of the following irradiating (influx) and reflection/transmission (efflux) conditions [VII].

5.1 Recommended nomenclature for directional irradiation

5.1.1 Forty-five degree directional geometry (45°x)

Irradiation of reflecting materials at 45° to the normal, at one azimuth angle, emphasizes texture and directionality. The "*x*" in the symbol indicates that the azimuthal direction of the incident beam is in the *x* direction on the reference plane.

5.1.2 Forty-five degree annular geometry (45°a)

In measuring the colours of reflecting samples with 45° illumination, the effects of texture and directionality are minimized by irradiating at 45° to the normal, from all azimuthal directions, simultaneously. This irradiation (influx) geometry may be achieved by the use of a small source and an elliptic ring reflector or other aspheric optics. This geometry is sometimes approximated by the use of a number of light sources in a ring or a number of fibre bundles illuminated by a single source and terminated in a ring. Such an approximation to annular geometry is called *circumferential geometry*, symbol 45°*c*.

5.1.3 Zero degree directional geometry (0°)

Irradiation of reflecting materials at the normal.

[VII] Users of this report should check with the CIE list of publications for a technical report on this subject.

5.1.4 Eight degree geometry (8°)

Irradiation of reflecting materials at 8° to the normal, at one azimuth angle. It is used to substitute the zero-degree directional geometry in many practical applications, as in reflection measurements it permits differentiation between specular component included and excluded measurements.

5.2 Recommended geometry for reflection measurements

5.2.1 Diffuse: eight-degree geometry, specular component included (di:8°)

For di:8° geometry, it is recommended that the sample be irradiated by an integrating sphere[8], that the measured area be overfilled, that the area of the sampling aperture be uniformly irradiated, that it be irradiated uniformly from all directions within the hemisphere bounded by its plane, that the responsivity of the receiver be uniform over the area of the sampling aperture, that the axis of the reflected (efflux) beam be 8° off the normal to the centre of the sample, and that radiation reflected at the sampling aperture be evaluated uniformly at all directions within 5° of the axis of the collection (efflux) beam. The size of the sampling aperture, area and angular uniformity of irradiation, angular displacement of the collected (efflux) beam from the normal, and area and angular uniformity of responsivity of the receiver can affect measured values and may be standardized in the future.

5.2.2 Diffuse: eight-degree geometry, specular component excluded (de:8°)

It is recommended that the specifications for di:8° be met, except that there be no radiation reflected in the direction of the receiver by a plane first-surface mirror at the sampling aperture and that there be no rays specularly reflected within 1° of such rays, as an allowance for instrumental scattering of stray light or misalignment. The amount of stray light specularly reflected in the direction of the receiver may affect measured values and may be standardized in the future.

5.2.3 Eight degree: diffuse geometry, specular component included (8°:di)

It is recommended that the conditions for di:8° be met, but with the light path reversed, so the sampling aperture is irradiated under 8° and flux reflected at the sampling aperture is collected at all angles in the hemisphere bounded by the reference plane. The sampling aperture should be underfilled with radiation.

5.2.4 Eight degree: diffuse geometry, specular component excluded (8°:de)

It is recommended that the geometric conditions for de:8° be met, but with the light path reversed. The sampling aperture should be underfilled with radiation.

5.2.5 Diffuse / diffuse geometry (d:d)

It is recommended that for irradiation the specifications for di:8° be met, and flux reflected at the sampling aperture is collected at all angles in the hemisphere bounded by the reference plane. In this geometry the sampling aperture can be neither underfilled nor overfilled.

5.2.6 Alternative diffuse geometry (d:0°)

An alternative diffuse geometry is the strict specular excluded geometry defined when the efflux direction is along the specimen normal.

5.2.7 Forty-five degree annular / normal geometry (45°a:0°)

It is recommended that the sampling aperture be irradiated uniformly from all directions between two right circular cones with their axes on the normal to the sampling aperture and apices at the centre of the sampling aperture, the smaller cone having a half angle of 40° and the larger of 50°. It is recommended that the receiver uniformly collects and evaluates all radiation reflected within a cone with its axis on the normal to the sampling aperture, apex at the centre of the sampling aperture, and a half angle of 5°. If this illuminating geometry is approximated by the use of a number of light sources in a ring or a number of fibre bundles illuminated by a single source and terminated in a ring, one gets the circumferential / normal geometry (45°c:0°). The above angular specifications should also hold for this geometry. These angular specifications and the surface and angular uniformity of irradiation and sensing may affect measured values and may be standardized in the future.

5.2.8 Normal / forty-five degree annular geometry (0°:45°a)

It is recommended that the angular and spatial conditions for 45°a:0° be met, with the light path reversed, so the sampling aperture is irradiated normally and reflected radiation is collected within an annulus centred at 45° to the normal.

5.2.9 Forty-five degree directional / normal geometry (45°x:0°)

It is recommended that the angular and spatial conditions for 45°a:0° be met with the exception that irradiation is only at one azimuth angle, this excludes the specular component, but emphasizes texture and directionality. The "*x*" in the symbol indicates that the azimuthal direction of the incident beam is in the *x* direction on the reference plane.

5.2.10 Normal / forty-five degree directional geometry (0°:45°x)

It is recommended that the angular and spatial conditions for 45°x:0° be met, with the light path reversed, so the sampling aperture is irradiated normally and reflected radiation is collected at one azimuth angle at 45° to the normal.

Notes on recommended geometry for reflection measurements

Note 1: Conditions 5.2.1, 5.2.2, 5.2.6, 5.2.7, 5.2.8, 5.2.9 and 5.2.10 give values of reflectance factor, $R(\lambda)$. For directional measurement with a sufficiently small angular spread, these values of reflectance factor become identical to values of radiance factor. For condition 5.2.3, for measuring with an ideal sphere, reflectance is measured. Thus, in the limit, the 45°x:0° condition gives the radiance factor $\beta_{45:0}$; the 0°:45°x condition gives the radiance factor $\beta_{0:45}$; the di:8° condition gives the factor $\beta_{di:8}$; that approximates the radiance factor $\beta_{d:0}$; and the 8°:di condition gives the reflectance ρ (see CIE, 1987).

Note 2: It is important that the particular irradiating and measuring conditions used should be specified even if they are within the range of one of these recommended standard conditions. Measurements of some types of samples (for example retro-reflective materials) may require different geometry or tolerances.

Note 3: When integrating spheres are used, they should be fitted with white-coated baffles to prevent light passing directly between the sample and the spot of the sphere wall irradiated or measured. When the regular component of reflection is to be included, the sphere efficiency for that part of sphere wall that receives the regularly reflected radiation component should be of the same reflectance value as the sphere wall. The total area of the ports of the integrating sphere should not exceed 10 percent of the internal reflecting sphere area.

Note 4: It should be noted that diffusing samples may scatter radiation in directions approximately parallel to their surfaces, and such radiation should be included in the measurement of diffuse reflectance.

Note 5: When integrating spheres are used for measuring luminescent samples, the spectral power distribution of the irradiating system is altered by the reflected and emitted power from the sample (see CIE, 1988). The use of the 45°a:0°, 45°x:0° or 0°:45°a, 0°:45°x condition is therefore preferable (see Gundlach and Mallwitz, 1976; Alman and Billmeyer, 1976).

5.3 Recommended geometry for transmission measurements

5.3.1 Normal / normal geometry (0°:0°)

It is recommended that the irradiating (influx) and measuring (efflux) geometry be of identical right-circular conic form, with their axes on the normal to the centre of the sampling aperture, and half-angle of 5°, that the surface and angular irradiation of the sampling aperture be uniform, and that the surface and angular responsivity of the receiver be uniform. Deviation of axes from normal and variations in surface and angular conditions may affect measured values and may be standardized in the future.

5.3.2 *Diffuse / normal geometry, regular component included (di:0°)*

It is recommended that the sampling aperture be uniformly irradiated from all directions in the hemisphere bounded by the first reference plane and that the measuring (efflux) beam be as specified for 0°:0° geometry. Deviation of the measuring (efflux) axis from normal and variations in surface and angular conditions may affect measured values and may be standardized in the future.

5.3.3 *Diffuse / normal geometry, regular component excluded (de:0°)*

It is recommended that the geometry be that specified for di:0° except that, with the sampling aperture open (i.e. no sample in place), there be no rays directed toward the receiver and no rays within 1° of such rays, as measured at the centre of the sampling aperture.

5.3.4 *Normal / diffuse geometry, regular component included (0°:di)*

It is recommended that the geometry be the reverse of that specified for di:0° geometry.

5.3.5 *Normal / diffuse geometry, regular component excluded (0°:de)*

It is recommended that the geometry be the reverse of that specified for de:0° geometry.

5.3.6 *Diffuse / diffuse geometry (d:d)*

It is recommended that the sampling aperture be uniformly irradiated at all angles within the hemisphere bounded by the first reference plane and that the transmitted flux be uniformly evaluated at all directions within the hemisphere bounded by the second reference plane.

<u>Notes on recommended geometry for transmission measurements</u>

Note 1: All the above conditions measure transmittance except for those where the regular component is excluded when the quantity measured is transmittance factor.

Note 2: It is important that the particular irradiating and collecting conditions used are specified, even if they are within the range of one of the conditions recommended here. Measurements of some types of samples may require different geometry or tolerances.

Note 3: Integrating spheres shall be fitted with white-coated baffles to prevent radiation passing directly from source to sample or reference in the case of diffuse irradiation or directly from sample or reference to detector in the case of diffuse collection. The total area of the ports of the integrating sphere should not exceed 10 percent of the internal reflecting sphere area.

Note 4: The construction of an instrument for normal/normal measurements shall be such that the irradiating (influx) and collecting (efflux) beams shall be equal whether there is a sample in place or not.

Note 5: It should be noted that diffusing samples may scatter radiation in directions approximately parallel to their surfaces, and such radiation should be included in the measurement of diffuse transmittance.

Note 6: Multiple reflections between the sample and the incident beam optics if the incident beam is normal to the sample surface may cause measurement errors. These can be eliminated by slightly tilting the sample.

6. RECOMMENDATIONS CONCERNING STANDARD OBSERVER DATA[9]

6.1 CIE 1931 standard colorimetric observer

For correlation with visual colour matching of fields subtending between about 1° and about 4° at the eye of the observer, it is recommended that colorimetric specifications of colour stimuli be based on the colour-matching functions $\bar{x}(\lambda), \bar{y}(\lambda), \bar{z}(\lambda)$[10]. A 2° visual field represents a diameter of about 17 mm at a viewing distance of 0,5 m. These colour-matching functions are given in the standard as values from 360 nm to 830 nm at 1 nm intervals with seven significant figures, and they define the CIE 1931 standard colorimetric observer (in technical applications often written as 2°-standard colorimetric observer)[VIII]. In the case where more coarsely sampled data will produce no significant calculation error, the tables reproduced in Section 11 may be used instead of the 1 nm interval data. The values given in Table T.4 at 5 nm intervals are selected values from the standard, rounded to six decimal places[11].

Note 1: $\bar{x}(\lambda), \bar{y}(\lambda), \bar{z}(\lambda)$ are the normalized tristimulus values of monochromatic radiations for a set of reference stimuli [X], [Y], [Z] required to match each wavelength of the equi-energy spectrum. Their levels are such that the maximum of $\bar{y}(\lambda)$ is unity and $\Sigma\bar{x}(\lambda) = \Sigma\bar{z}(\lambda) = \Sigma\bar{y}(\lambda)$. (The equi-energy spectrum is radiation whose spectral concentration of power as a function of wavelength is constant.)

Note 2: The real reference stimuli [R], [G], [B] of the original trichromatic system lead, by transformation, to [X], [Y], [Z]. $\bar{y}(\lambda)$ is equated to $V(\lambda)$, the spectral luminous efficiency function for photopic vision, defining the CIE standard photometric observer for photopic vision (for more details see Appendix B).

Note 3: The reference stimuli [X], [Y], [Z] were chosen for reasons of convenience in colorimetric computations. The colour-matching functions $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, $\bar{z}(\lambda)$ are commonly used to obtain the tristimulus values X, Y, Z of colour stimuli (see Section 7.1 for details).

Note 4: If the colour-matching functions at closer intervals than given in Table T.4 are required, the values given in the CIE Standard S002 (CIE, 1986a) should be used. For interpolation at wavelength intervals smaller than 1 nm a linear interpolation should be used.

Note 5: The chromaticity coordinates (see Section 7.3) $x(\lambda)$, $y(\lambda)$, $z(\lambda)$ of the spectral stimuli are the ratios

$$\begin{aligned} x(\lambda) &= \frac{\bar{x}(\lambda)}{\bar{x}(\lambda)+\bar{y}(\lambda)+\bar{z}(\lambda)} \\ y(\lambda) &= \frac{\bar{y}(\lambda)}{\bar{x}(\lambda)+\bar{y}(\lambda)+\bar{z}(\lambda)} \\ z(\lambda) &= \frac{\bar{z}(\lambda)}{\bar{x}(\lambda)+\bar{y}(\lambda)+\bar{z}(\lambda)} \end{aligned} \tag{6.1}$$

$x(\lambda)$ and $y(\lambda)$ are given in Table T.4.

Note 6: The chromaticity coordinates x_E, y_E, z_E of the equi-energy spectrum derived from the sums $\Sigma\bar{x}(\lambda), \Sigma\bar{y}(\lambda), \Sigma\bar{z}(\lambda)$ of Table T.4 are

$$\begin{aligned} x_E &= 0{,}333\,334 \\ y_E &= 0{,}333\,331 \\ z_E &= 0{,}333\,335 \end{aligned} \tag{6.2}$$

The small differences between the values of the chromaticity coordinates x_E, y_E, z_E are due to the limited number of decimal digits given in the table.

6.2 CIE 1964 standard colorimetric observer

For correlation with visual colour matching of fields of angular subtense greater than 4° at the eye of the observer, it is recommended that colorimetric specifications of colour stimuli be

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[VIII] Table 1 of CIE standard colorimetric observers (CIE, 1986a) is reproduced in the CD-ROM that accompanies this Technical Report.

based on the colour-matching functions $\bar{x}_{10}(\lambda), \bar{y}_{10}(\lambda), \bar{z}_{10}(\lambda)$ published in the CIE Standard: CIE standard colorimetric observers (CIE, 1986a). A 10° visual field represents a diameter of about 90 mm at a viewing distance of 0,5 m. These colour-matching functions are given in the standard as values from 360 nm to 830 nm at 1 nm intervals with six significant figures, and they define the CIE 1964 standard colorimetric observer (in technical applications often written as 10°-standard colorimetric observer)[IX]. In the case more coarsely sampled data will produce no significant calculation error, the tables reproduced in Section 11 may be used instead of the 1-nm interval data. The values given in Table T.5 at 5 nm intervals are selected values from the standard, rounded to six decimal places.

Note 1: $\bar{x}_{10}(\lambda), \bar{y}_{10}(\lambda), \bar{z}_{10}(\lambda)$ are the normalized tristimulus values of monochromatic radiations for a set of reference stimuli [X_{10}], [Y_{10}], [Z_{10}] required to match each wavelength of the equi-energy spectrum. Their levels are such that the maximum of $\bar{y}_{10}(\lambda)$ is unity and $\Sigma\bar{x}_{10}(\lambda) = \Sigma\bar{y}_{10}(\lambda) = \Sigma\bar{z}_{10}(\lambda)$ (The equi-energy spectrum is radiation whose spectral concentration of power as a function of wavelength is constant.)

Note 2: The real reference stimuli [R_{10}], [G_{10}], [B_{10}] of the original trichromatic system are related by transformation to the reference stimuli [X_{10}], [Y_{10}], [Z_{10}], which were chosen for reasons of convenience in colorimetric computations (see Appendix B). The colour-matching functions, $\bar{x}_{10}(\lambda), \bar{y}_{10}(\lambda), \bar{z}_{10}(\lambda)$ are commonly used to obtain the tristimulus values X_{10}, Y_{10}, Z_{10} of colour stimuli.

Note 3: If the colour-matching functions of Table T.5 are required at closer intervals, the values at 1 nm intervals given in the standard, CIE standard colorimetric observers (CIE, 1986a) should be used. For values at other than the 1 nm intervals see Note 4 to 6.1.

Note 4: The chromaticity coordinates $x_{10}(\lambda)$, $y_{10}(\lambda)$, $z_{10}(\lambda)$ of the spectral stimuli are the ratios

$$\begin{aligned} x_{10}(\lambda) &= \frac{\bar{x}_{10}(\lambda)}{\bar{x}_{10}(\lambda) + \bar{y}_{10}(\lambda) + \bar{z}_{10}(\lambda)} \\ y_{10}(\lambda) &= \frac{\bar{y}_{10}(\lambda)}{\bar{x}_{10}(\lambda) + \bar{y}_{10}(\lambda) + \bar{z}_{10}(\lambda)} \\ z_{10}(\lambda) &= \frac{\bar{z}_{10}(\lambda)}{\bar{x}_{10}(\lambda) + \bar{y}_{10}(\lambda) + \bar{z}_{10}(\lambda)} \end{aligned} \tag{6.3}$$

$x_{10}(\lambda)$ and $y_{10}(\lambda)$ are given in Table T.5.

Note 5: The chromaticity coordinates $x_{10,\mathrm{E}}$, $y_{10,\mathrm{E}}$, $z_{10,\mathrm{E}}$ of the equi-energy spectrum derived from the sums $\Sigma\bar{x}_{10}(\lambda)$, $\Sigma\bar{y}_{10}(\lambda)$, $\Sigma\bar{z}_{10}(\lambda)$ of Table T.5 are

$$\begin{aligned} x_{10,\mathrm{E}} &= 0{,}333\,298 \\ y_{10,\mathrm{E}} &= 0{,}333\,336 \\ z_{10,\mathrm{E}} &= 0{,}333\,366 \end{aligned} \tag{6.4}$$

The small differences between the values of the chromaticity coordinates $x_{10\mathrm{E}}$, $y_{10\mathrm{E}}$, $z_{10\mathrm{E}}$ are due to the limited number of decimal digits given in the table.

Note 6. The large-field colour matching data as defined by the CIE 1964 standard colorimetric observer are intended to apply to matches where the luminance and the relative spectral power distributions of the matched stimuli are such that no participation of the rod receptors of the visual mechanism is to be expected. This condition of observation is important as "rod intrusion" may upset the predictions of the standard observer. For daylight, possible participation of rod vision in colour matches is likely to diminish progressively above about 10 cd·m^{-2} and be entirely absent at about 200 cd·m^{-2}. For further details see [12].

[IX] Table 2 of CIE standard colorimetric observers (CIE, 1986a) is reproduced on the CD-ROM that accompanies this Technical Report.

The CIE 1931 and 1964 standard colorimetric observers were both derived from trichromatic systems based on real red [R], green [G] and blue [B] primaries, see Appendix B.

7. RECOMMENDATIONS CONCERNING THE CALCULATION OF TRISTIMULUS VALUES AND CHROMATICITY COORDINATES[13]

7.1 Calculation of tristimulus values

The CIE Standard (CIE, 1986a) on standard colorimetric observers recommends that the CIE tristimulus values of a colour stimulus be obtained by multiplying at each wavelength the value of the colour stimulus function $\phi_\lambda(\lambda)$ by that of each of the CIE colour-matching functions and integrating each set of products over the wavelength range corresponding to the entire visible spectrum, 360 nm to 830 nm. The integration can be carried out by numerical summation at wavelength intervals, $\Delta\lambda$, equal to 1 nm.

$$\begin{aligned} X &= k\sum_\lambda \phi_\lambda(\lambda)\bar{x}(\lambda)\Delta\lambda & \qquad X_{10} &= k_{10}\sum_\lambda \phi_\lambda(\lambda)\bar{x}_{10}(\lambda)\Delta\lambda \\ Y &= k\sum_\lambda \phi_\lambda(\lambda)\bar{y}(\lambda)\Delta\lambda & \qquad Y_{10} &= k_{10}\sum_\lambda \phi_\lambda(\lambda)\bar{y}_{10}(\lambda)\Delta\lambda \\ Z &= k\sum_\lambda \phi_\lambda(\lambda)\bar{z}(\lambda)\Delta\lambda & \qquad Z_{10} &= k_{10}\sum_\lambda \phi_\lambda(\lambda)\bar{z}_{10}(\lambda)\Delta\lambda \end{aligned} \qquad (7.1)$$

In the above equations $\phi_\lambda(\lambda)$ denotes the spectral distribution of the colour stimulus function, i.e. $\phi_\lambda(\lambda) = \mathrm{d}\phi(\lambda)/\mathrm{d}\lambda$, see CIE International Lighting Vocabulary item 845-01-17 (CIE, 1987). X, Y, Z are tristimulus values, $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, $\bar{z}(\lambda)$ are colour-matching functions of a standard colorimetric observer, and k is a normalising constant defined below. Each of these may be specified for the CIE 1931 standard colorimetric system by being written without a subscript, or for the CIE 1964 standard colorimetric system by the use of the subscript 10.

The fundamental colorimetric tables are the 1 nm tables in CIE standards. All rigorous calculations should use these 1 nm tables. For most practical purposes, the summation may be approximated by using wavelength intervals, $\Delta\lambda$ equal to 5 nm over the wavelength range 380 nm to 780 nm. Values of the CIE colour-matching functions at 5 nm intervals suitable for use in summation over this range of wavelengths are given in Tables T.4 and T.5. When measurements have been made at smaller intervals than 5 nm, the appropriate values from the tables in the standards should be used.

Tristimulus values are usually evaluated on a relative basis. In such cases the relative colour stimulus function, $\phi(\lambda)$, may be used instead of the colour stimulus function, $\phi_\lambda(\lambda)$, but it is essential that, for stimuli that will be considered together, all the spectral distributions involved be assessed on the same relative basis. The tristimulus values obtained are then relative in the sense that all the values involved may be multiplied by the same single arbitrary constant. In certain cases, however, k and k_{10} must be chosen according to agreed conventions; these are explained in Sections 7.1.1 and 7.1.2.

Tristimulus values evaluated on a relative basis may need to be supplemented by the value of a suitable absolute photometric quantity.

7.1.1 Secondary light sources (reflecting or transmitting objects)

For reflecting or transmitting object colours, the colour stimulus function, $\phi_\lambda(\lambda)$, is replaced by the relative colour stimulus function, $\phi(\lambda)$, evaluated as

$$\phi(\lambda) = R(\lambda)\cdot S(\lambda) \qquad \text{or} \qquad \phi(\lambda) = \tau(\lambda)\cdot S(\lambda)$$

where: $R(\lambda)$ is the spectral reflectance factor (or spectral radiance factor or spectral reflectance) of the object colour (preferably evaluated for one of the geometric conditions given in Section 5.2).

$\tau(\lambda)$ is the spectral transmittance of the object colour (preferably evaluated for one of the geometric conditions given in Section 5.3).

$S(\lambda)$ is the relative spectral power distribution of the illuminant (which, whenever possible, should be one of the CIE standard illuminants; see Section 3.1).

In this case, the constants, k and k_{10}, are chosen so that $Y = 100$ for objects for which $R(\lambda)$, or $\tau(\lambda) = 1$ for all wavelengths, and hence

$$k = 100 / \sum_{\lambda} S(\lambda)\bar{y}(\lambda)\Delta\lambda$$

$$k_{10} = 100 / \sum_{\lambda} S(\lambda)\bar{y}_{10}(\lambda)\Delta\lambda \tag{7.2}$$

For other objects, the values of Y are then equal to the percentage values of luminous reflectance factor or luminance factor or luminous reflectance [in the case of $R(\lambda)$], or luminous transmittance or luminous transmittance factor [in the case of $\tau(\lambda)$]; this is because the $\bar{y}(\lambda)$ function is identical to the CIE spectral luminous efficiency function $V(\lambda)$[X].

7.1.2 *Illuminants and self-luminous objects*

For self-luminous objects and illuminants, the constants, k and k_{10}, are usually chosen on the grounds of convenience. If, however, in the CIE 1931 standard colorimetric system, the Y value is required to be numerically equal to the absolute value of a photometric quantity, the constant, k, must be put equal to the numerical value of K_m, the maximum spectral luminous efficacy (which is equal to 683 $\text{lm}\cdot\text{W}^{1}$) and $\phi_\lambda(\lambda)$ must be the spectral concentration of the radiometric quantity corresponding to the photometric quantity required.

7.2 The use of abridged or truncated data

The use of the tables published in the CIE standard on colorimetric observers (CIE, 1986a) requires that the colour stimulus function, $\phi_\lambda(\lambda)$, or the relative colour stimulus function, $\phi(\lambda)$, be known at 1 nm wavelength intervals over the wavelength range 360 nm to 830 nm. In practical applications, all the required data may not be available because the measurement was made at intervals greater than 1 nm, or unequal wavelength intervals were used, or data at the spectral extremes were omitted (truncation). Often it is possible to predict the needed but unmeasured data. It is recognised that calculation from predicted, rather than from measured, data, or from abridged or truncated data, may be inexact, but the investigator can often be satisfied that the accuracy is sufficient for the intended use of the tristimulus values. It is important to use the same wavelength interval and range throughout for any set of calculations in which data for different colours are to be compared precisely. In particular, it should be noted that the exact values obtained for the perfect diffuser for a given illuminant depend on the wavelength interval and range used[14].

7.2.1 *Abridgement*

Abridgement of colour stimulus data by the use of a larger sampling interval, for example 10 nm or 20 nm, will lead to errors in the computed tristimulus values. Such abridgement should be used only when it can be demonstrated that these errors are negligibly small for the intended use of the tristimulus values. If these errors are not negligible, *it is recommended that needed but unmeasured values of* $\phi_\lambda(\lambda)$, $R(\lambda)$, or $\tau(\lambda)$ *be predicted by adequate interpolation.*

Note 1: Spectral reflectance and transmittance curves are usually smooth enough and continuous to permit interpolation, while e.g. spectral power distribution data of gas discharge lamps are frequently not suited for interpolation. Using interpolated values of spectral power distribution data of such lamps may lead to considerable errors.

[X] CIE Division 1 is currently investigating whether a $V_{10}(\lambda)$ function should be introduced, where $V_{10}(\lambda) = \bar{y}_{10}(\lambda)$. Interested parties should check the latest CIE publication list or the recent edition of the International Lighting Vocabulary on the introduction of such a new definition. At the closing of the present edition of CIE 15 no $V_{10}(\lambda)$ function has been officially agreed (for further details see Appendix B).

Note 2: Measurement errors also arise from the bandwidth of a spectrometer. Even if the data interval is 1 nm, the colorimetric errors can be significant if the bandwidth of the spectrometer is large. See 7.2.5 for bandwidth requirements.

7.2.1.1 Interpolation

Use one of the four following methods to calculate needed but unmeasured values of $\phi_\lambda(\lambda)$, $R(\lambda)$ or $\tau(\lambda)$ within the range of measurements: 1) the third-order polynomial interpolation (Lagrange) from the four neighbouring data points around the point to be interpolated, or 2) cubic spline interpolation formula, or 3) a fifth order polynomial interpolation formula from the six neighboring data points around the point to be interpolated, or 4) a Sprague interpolation (see Sève, 2003)[XI]

Note: Experiments have shown that for most samples showing smooth reflectance or transmittance spectra, the computed tristimulus values will have an adequate accuracy if measured data, illuminant and colour-matching function values at 5 nm intervals are used. See also Note to Section 7.2.3.

7.2.2 *Truncation*

Where the measurement range is less than the practical range of summation, 380 nm to 780 nm, omission of values at the limits of the measurement range will lead to errors in the computed tristimulus values. Such truncation should be used only when it can be demonstrated that these errors are negligibly small for the intended use of the tristimulus values.

Note: If these errors due to limited measurement range are not negligibly small, the errors *might be* reduced by adequate extrapolation of the needed but unmeasured values of $\phi_\lambda(\lambda)$, $R(\lambda)$ or $\tau(\lambda)$. The range of the summation is an essential part of the tristimulus specification.

7.2.2.1 Extrapolation

Extrapolation is generally not recommended. However, when predicting needed but unmeasured values of $\phi_\lambda(\lambda)$ or $\phi(\lambda)$ outside the range of measurements, in the absence of other information, unmeasured values of $\phi_\lambda(\lambda)$, $[\phi(\lambda)]$ or of $\rho(\lambda)$, $\beta(\lambda)$, or $\tau(\lambda)$ may, as a rough approximation, be set equal to the nearest measured value of the appropriate quantity in truncation.

7.2.3 *Weighting factors*

In the case of repetitive calculations from measurements at the same wavelengths, it is often convenient to perform only once those calculations that are independent of the measured data. This results in a table of weighting factors. Such weighting factors should give results consistent with those from the normal calculations. It is recommended that tables of weighting factors be calculated for the full range of wavelengths, 360 nm to 830 nm. This provides tables that may be used for any degree of truncation by adding the weights at the unmeasured wavelengths to those at the extreme measured wavelengths. The sum of the weights at the unmeasured wavelengths is a measure of the maximum error introduced by truncation.

Note: If it has been checked that measurement data obtained only at 10 nm or 20 nm intervals satisfy the need of the observer, computation methods as described e.g. in ASTM (1999) might be used. This publication contains weighting factors for both the CIE 1931 standard colorimetric observer and the CIE 1964 standard colorimetric observer and a number of illuminants and practical light sources used in colorimetry. Its Table 5 has been developed for the case when the instrument manufacturer has built in a correction to zero bandwidth (see Fairman, 1985). Its Table 6 provides

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[XI] A CIE Technical Committee is currently working on a standardized method of interpolation, interested parties should check the latest CIE publication list for more information. The CD-ROM accompanying this report contains PC interpolation programs for linear, Lagrange, spline and Sprague interpolation.

weighting factors for the case when a correction to a zero bandwidth is required (see also Venable, 1989; Li et al., 2004; ASTM, 2001 and ASTM, 2003).

7.2.4 Numerical procedures

It is recommended that all numerical calculations be carried out using the full number of significant digits provided by the data in the Tables published in the CIE standards of colorimetry. Final results should be rounded to the number of significant digits indicated by the precision of the measurements.

Note 1: For the purpose of calculating tristimulus values, the quantities involved for secondary light sources shall be defined as points of a continuous distribution, each point representing an infinitely narrow range of wavelengths. For primary light sources the relative emitted power is regarded as emitted in a wavelength band equal to the measuring interval and centred around the nominal wavelength.

Note 2: For colour stimulus functions of primary light sources that vary rapidly with wavelength, the spectral bandwidth at half power of the measuring instrument used to determine the colour stimulus function should be equal to, or an integer multiple of, the measuring interval. For smoothly varying functions this restriction is less important. For the highest accuracy, a bandwidth of 1 nm may be used for the measuring instrument, but for most practical purposes a bandwidth and measurement interval of 5 nm may be used. The use of bandwidths of 10 nm, or 20 nm is not recommended; it can lead to considerable loss of accuracy, and if applied, should be checked on typical spectra. For practical measurements a trade-off between bandwidth and signal-to-noise ratio has to be found, as the latter will influence photometric accuracy. A signal-to-noise ratio of $1{:}10^{-4}$ when measuring an ideal white sample is adequate for most applications.

Note 3: The foregoing recommendations are based on measurements at equal intervals of wavelength. It is, however, not intended to imply that summations may properly be computed only with such intervals. Other summation procedures, including specifically the method of selected ordinates, may be used providing that the tristimulus values thus computed are consistent with those based on the above recommendations.

7.2.5 Bandwidth of a spectrometer

When discussing errors associated with data intervals, it is also important to know measurement errors (uncertainties) due to the bandwidth of a spectrometer because, in real measurements, the data interval (scanning interval) and the bandwidth are closely related with each other to affect the measurement results.

Measurement errors arising from the bandwidth of a spectrometer are generally much larger (by an order of magnitude) than the calculation errors associated with data intervals. Even if the data interval is 1 nm (or interpolated to 1 nm intervals), the colorimetric errors can be significant if bandwidth of the spectrometer is large. See also Section 7.2.4 Note 2.

7.3 Calculation of chromaticity coordinates

It is recommended that chromaticity coordinates (x, y, z) be derived from the tristimulus values (X, Y, Z) as follows:

$$x = \frac{X}{X+Y+Z} \qquad y = \frac{Y}{X+Y+Z} \qquad z = \frac{Z}{X+Y+Z} \tag{7.3}$$

Because of the relation $x + y + z = 1$, it suffices to quote x, y only. The diagram using the chromaticity coordinates x, y is referred to as the CIE 1931 chromaticity diagram or the CIE (x, y) (chromaticity) diagram.

Note: The chromaticity coordinates x_{10}, y_{10}, z_{10} are computed similarly from the tristimulus values X_{10}, Y_{10} , Z_{10}; the CIE 1964 chromaticity diagram is obtained using these values.

7.4 Equations representing relationships between colour stimuli [16]

When equations are used to represent relationships between colour stimuli, symbols of vector notation should be used instead of those for numerical relationships. For example, one of the following forms could be used:

$$[\mathrm{C}] \equiv X[\mathrm{X}] + Y[\mathrm{Y}] + Z[\mathrm{Z}]$$

or (7.4)

$$\mathbf{C} = X\mathbf{X}+Y\mathbf{Y}+Z\mathbf{Z}$$

where X, Y, Z are the tristimulus values of colour stimulus [C], or **C**. The unit vectors of the reference stimuli are indicated either by [X], [Y], [Z], or by the boldface Roman letters **X**, **Y**, **Z**.
Note: In Equ. 7.4 the "≡" and "=" signs mean "matches".

8. RECOMMENDATIONS CONCERNING UNIFORM COLOUR SPACING AND COLOUR DIFFERENCES[16]

8.1 CIE 1976 uniform chromaticity scale diagram (UCS diagram)

The use of the following chromaticity diagram is recommended whenever a projective transformation of the (x,y)-diagram yielding colour spacing perceptually more uniform than that of the (x,y)-diagram is desired. The chromaticity diagram is produced by plotting

$$u' = 4X / (X + 15Y + 3Z)$$

as abscissa and (8.1)

$$v' = 9Y / (X + 15Y + 3Z)$$

as ordinate, in which X, Y, Z are tristimulus values. The third chromaticity coordinate w' is equal to $(1 - u' - v')$.

Note 1: The colour spacing afforded by this chromaticity diagram is known to be perceptually more uniform than that of the CIE (x,y)-chromaticity diagram for observation of samples having negligibly different luminances (e.g. for $\Delta Y<0{,}5$). This diagram is intended to apply to comparisons of differences between object colours of the same size and shape, viewed in identical white to middle-grey surroundings, by an observer photopically adapted to a field of chromaticity not too different from that of average daylight.

Note 2: The same chromaticity diagram is produced by plotting

$$u' = 4x / (-2x + 12y + 3) \text{ as abscissa and}$$
$$v' = 9y / (-2x + 12y + 3) \text{ as ordinate,} \qquad (8.2)$$

where x, y are chromaticity coordinates.

Note 3: If the angle subtended at the eye by the pairs of specimens being compared is more than 1° and less than about 4°, the tristimulus values X, Y, Z (or chromaticity coordinates x, y), calculated using the CIE 1931 standard colorimetric observer should be used for the calculation of u' and v'. If the angle is greater than 4°, the tristimulus values X_{10}, Y_{10}, Z_{10}, (or chromaticity coordinates x_{10}, y_{10}) of the CIE 1964 standard colorimetric observer should be used to calculate u'_{10} and v'_{10}.

Note 4: The CIE 1960 UCS diagram, now obsolete, is described briefly in Appendix A.3.

8.2 CIE 1976 uniform colour spaces

Pending the development of an improved coordinate system[17], the use of one of the following coordinate systems is recommended whenever a three-dimensional spacing perceptually more nearly uniform than that provided by the XYZ system is desired.

*8.2.1 CIE 1976 (L*a*b*) colour space; CIELAB colour space*

8.2.1.1 Basic coordinates

Three-dimensional, approximately uniform, colour space produced by plotting in rectangular coordinates, L^*, a^*, b^*, quantities defined by the equations

$$L^* = 116\, f(Y/Y_n) - 16 \tag{8.3}$$

$$a^* = 500[f(X/X_n) - f(Y/Y_n)] \tag{8.4}$$

$$b^* = 200[f(Y/Y_n) - f(Z/Z_n)] \tag{8.5}$$

where

$$f(X/X_n) = (X/X_n)^{1/3} \quad \text{if} \quad (X/X_n) > (24/116)^3 \tag{8.6}$$

$$f(X/X_n) = (841/108)(X/X_n) + 16/116 \quad \text{if} \quad (X/X_n) \leq (24/116)^3 \tag{8.7}$$

and

$$f(Y/Y_n) = (Y/Y_n)^{1/3} \quad \text{if} \quad (Y/Y_n) > (24/116)^3 \tag{8.8}$$

$$f(Y/Y_n) = (841/108)(Y/Y_n) + 16/116 \quad \text{if} \quad (Y/Y_n) \leq (24/116)^3 \tag{8.9}$$

and

$$f(Z/Z_n) = (Z/Z_n)^{1/3} \quad \text{if} \quad (Z/Z_n) > (24/116)^3 \tag{8.10}$$

$$f(Z/Z_n) = (841/108)(Z/Z_n) + 16/116 \quad \text{if} \quad (Z/Z_n) \leq (24/116)^3 \tag{8.11}$$

where X, Y, Z are the tristimulus values of the test object colour stimulus considered and X_n, Y_n, Z_n are the tristimulus values of a specified white object colour stimulus. In most cases, the specified white object colour stimulus should be light reflected from a perfect reflecting diffuser illuminated by the same light source as the test object. In this case, X_n, Y_n, Z_n are the tristimulus values of the light source with Y_n equal to 100[18].

The informative annex Appendix D describes the recommended reverse transformation of the L^*, a^*, b^* coordinates to X, Y, Z tristimulus values.

8.2.1.2 Correlates of lightness, chroma and hue

Approximate correlates of the perceived attributes lightness, chroma and hue are calculated as follows

CIE 1976 lightness: L^* as defined in Equ. 8.3.

CIE 1976 a,b (CIELAB) chroma : $C^*_{ab} = (a^{*2} + b^{*2})^{1/2}$ (8.12)

CIE 1976 a,b (CIELAB) hue angle: $h_{ab} = \arctan(b^*/a^*)$ (8.13)

see Section 8.2.3 Note 1 and Note 2.

8.2.1.3 Colour differences

Euclidean distances in CIELAB colour space can be used to represent approximately the perceived magnitude of colour differences between object colour stimuli of the same size and shape, viewed in identical white to middle-grey surroundings, by an observer photopically adapted to a field of chromaticity not too different from that of average daylight. In cases of deviating conditions the correlation between calculated and perceived colour differences may be impaired.

Differences between two samples (denoted by subscripts 0 and 1) shall be calculated as follows:

CIELAB lightness difference: $\Delta L^* = L^*_1 - L^*_0$ (8.14)

$\Delta a^* = a^*_1 - a^*_0$ (8.15)

$\Delta b^* = b^*_1 - b^*_0$ (8.16)

CIELAB chroma difference: $\Delta C^*_{ab} = C^*_{ab,1} - C^*_{ab,0}$ (8.17)

CIELAB hue angle difference: $\Delta h_{ab} = h_{ab,1} - h_{ab,0}$ (8.18)

If the line joining the two colours crosses the positive a^* axis, Equ. 8.18 will give a value outside the range ±180°, in this case, the value of Δh_{ab} must be corrected by adding or subtracting 360° to bring it within this range, see also Section 8.2.3 Note 4.

CIELAB hue difference $\Delta H^*_{ab} = 2(C^*_{ab,1} \cdot C^*_{ab,0})^{1/2} \cdot \sin(\Delta h_{ab}/2)$ (8.19)

for small colour differences away from the achromatic axis

$$\Delta H^*_{ab} = (C^*_{ab,1} \cdot C^*_{ab,0})^{1/2} \cdot \Delta h_{ab} \quad (8.20)$$

where the value of Δh_{ab} is in radians.

Note 1: The calculation of CIELAB hue and chroma differences is progressively less useful as the absolute value of Δh_{ab} approaches 180°.

Note 2: In information technology and other fields the subscript (R) is sometimes used for reference and (T) for test instead of (0) and (1). Similarly in industrial evaluation of small colour differences (s) is sometimes used for standard and (b) for batch.

CIE 1976 a,b (CIELAB) colour difference, ΔE^*_{ab} between two colour stimuli is calculated as the Euclidean distance between the points representing them in the space:

$$\Delta E^*_{ab} = \left[(\Delta L^*)^2 + (\Delta a^*)^2 + (\Delta b^*)^2\right]^{1/2} \quad (8.21)$$

or

$$\Delta E^*_{ab} = \left[\left(\Delta L^*\right)^2 + \left(\Delta C^*_{ab}\right)^2 + \left(\Delta H^*_{ab}\right)^2\right]^{1/2} \quad (8.22)$$

these two definitions of ΔE^*_{ab} are equivalent.

Alternative ways of calculating ΔH^*_{ab} are:

$$\Delta H^*_{ab} = \left[\left(\Delta E^*_{ab}\right)^2 - (\Delta L^*)^2 - \left(\Delta C^*_{ab}\right)^2\right]^{1/2} \quad (8.23)$$

where ΔE^*_{ab} is calculated from Equ. 8.21, and ΔH^*_{ab} has the same sign as Δh_{ab};

$$\Delta H^*_{ab} = k\left[2\left(C^*_{ab,1} \cdot C^*_{ab,0} - a^*_1 \cdot a^*_0 - b^*_1 \cdot b^*_0\right)\right]^{1/2} \quad (8.24)$$

where $k = -1$ if $a^*_1 \cdot b^*_0 > a^*_0 \cdot b^*_1$, otherwise $k = 1$;

and

$$\Delta H^*_{ab} = \left(a^*_0 \cdot b^*_1 - a^*_1 \cdot b^*_0\right) / \left[0{,}5\left(C^*_{ab,1} \cdot C^*_{ab,0} + a^*_1 \cdot a^*_0 + b^*_1 \cdot b^*_0\right)\right]^{1/2} \quad (8.25)$$

More details on these alternative methods of calculating ΔH^*_{ab} are given in Sève (1991), Stokes and Brill (1992) and Sève (1996).

Note: In different practical applications it may be necessary to use different weightings for ΔL^*, ΔC^*_{ab}, and ΔH^*_{ab}. In 2001, the CIE recommended such weightings in a new formula for industrial evaluation of small colour differences (CIE, 2001a), see Section 8.3.

*8.2.2 CIE 1976 (L*u*v*) colour space; CIELUV colour space*

Three-dimensional, approximately uniform, colour space produced by plotting in rectangular coordinates, L^*, u^*, v^*, quantities defined by the equations

$$L^* = 116\, f(Y/Y_n) - 16 \quad (8.26)$$

See also Equ. 8.3, where

$$f(Y/Y_n) = (Y/Y_n)^{1/3} \qquad \text{if} \quad (Y/Y_n) > \left(\frac{24}{116}\right)^3 \quad (8.27)$$

$$f(Y/Y_n) = (841/108)(Y/Y_n) + 16/116 \quad \text{if} \quad (Y/Y_n) \leq \left(\frac{24}{116}\right)^3 \tag{8.28}$$

and

$$u^* = 13L^*(u' - u'_n) \tag{8.29}$$

$$v^* = 13L^*(v' - v'_n) \tag{8.30}$$

where Y, u', v' describe the colour stimulus considered and Y_n, u'_n, v'_n describe a specified white object colour stimulus.

Approximate correlates of lightness, saturation, chroma, and hue may be calculated as follows:

CIE 1976 lightness: L^* as defined in Section 8.2.1.1 and Equ. 8.26.

CIE 1976 u,v (CIELUV) saturation: $$s_{u,v} = 13\,[(u' - u'_n)^2 + (v' - v'_n)^2]^{1/2} \tag{8.31}$$

CIE 1976 u,v (CIELUV) chroma: $$C^*_{uv} = (u^{*2} + v^{*2})^{1/2} = L^* s_{uv} \tag{8.32}$$

CIE 1976, u,v (CIELUV) hue-angle: $$h_{uv} = \arctan[(v' - v'_n)/(u' - u'_n)] = \arctan(v^*/u^*) \tag{8.33}$$

See 8.2.3, Note 2.

CIE 1976 u,v (CIELUV) hue-difference: $$\Delta H^*_{uv} = 2(C^*_{uv,1} \cdot C^*_{uv,0})^{1/2} \sin(\Delta h_{uv}/2) \tag{8.34}$$

where 1 and 0 refer to the two samples between which the colour difference is to be calculated and $\Delta h_{uv} = h_{uv,1} - h_{uv,0}$ (see Sève ,1991).

CIELUV colour difference ΔE^*_{uv} between two colour stimuli is calculated as the Euclidean distance between the points representing them in the space:

$$\Delta E^*_{uv} = [(\Delta L^*)^2 + (\Delta u^*)^2 + (\Delta v^*)^2]^{1/2} \tag{8.35}$$

For an alternative ways to calculate ΔH^*_{uv}, see the description in 8.2.1, but change a^* to u^* and b^* to v^*.

8.2.3 Notes on CIE 1976 uniform colour spaces

Note 1: When the linear formulae (Equ.'s 8.7, 8.9 or 8.11) are used for X/X_n, Y/Y_n or Z/Z_n, anomalous values of h_{ab} may be obtained (McLaren, 1980). Anomalous values are unlikely to occur for surface colours but may occur for transparent object colours of low luminance factor lying close to the spectrum locus or purple line.

Note 2: h_{ab} (or h_{uv}) lies between 0° and 90° if a^* and b^* (or u^* and v^*) are both positive, between 90° and 180° if b^* (v^*) is positive and a^* (u^*) is negative, between 180° and 270° if b^* and a^* (v^* and u^*) are both negative, and between 270° and 360° if b^* (v^*) is negative and a^* (u^*) is positive.

Note 3: CIE 1976 *a,b* and *u,v* hue-differences are introduced so that a colour difference ΔE^* can be broken up into components ΔL^*, ΔC^*, and ΔH^* whose squares sum to the square of ΔE^*. Differences in CIE 1976 *a,b* or *u,v* hue-angle, Δh_{ab} (or Δh_{uv}), do not have this property.

Note 4: If the line joining the two colours crosses the positive a^* (or u^*) axis, the value of Δh_{ab} (or Δh_{uv}) must be corrected by adding or subtracting 360° to bring it into the range ±180°.

Note 5: These spaces are intended to apply to comparisons of differences between object colours of the same size and shape, viewed in identical white to middle-grey surroundings, by an observer photopically adapted to a field of chromaticity not too different from that of average daylight.

Note 6: If the angle subtended at the eye by the pairs of object colours being compared is between about 1° and 4° the tristimulus values X, Y, Z calculated with respect to the CIE 1931 standard colorimetric observer should be used for the calculation of L^*, a^*, b^*, u^*, v^* and hence ΔE^*_{ab}, C^*_{ab}, s_{uv}, C^*_{uv}, h_{ab}, h_{uv}, ΔH^*_{ab}, ΔH^*_{uv}. If the angle is greater than 4°, the tristimulus values X_{10}, Y_{10}, Z_{10} calculated with respect to the CIE 1964 standard colorimetric observer should be used to calculate L^*_{10}, a^*_{10}, b^*_{10}, u^*_{10}, v^*_{10}, and hence $\Delta E^*_{ab,10}$, $C^*_{ab,10}$, $s_{uv,10}$, $C^*_{uv,10}$, $h_{ab,10}$, $h_{uv,10}$, $\Delta H^*_{ab,10}$, $\Delta H^*_{uv,10}$.

Note 7: The obsolete CIE 1964 uniform colour space and colour difference formula are described briefly in the Appendix A.

Note 8: In different practical applications it may be necessary to use different weightings for ΔL^*, ΔC^*, and ΔH^*, see Section 8.3.

Note 9: Equ.'s 8.21 to 8.25 and 8.35 have been replaced for small colour differences by the new recommendation of the CIE, see Section 8.3.

8.3 Improved industrial colour difference evaluation

8.3.1 CIEDE2000 total colour difference formula

The CIE 1976 uniform colour spaces provide for the calculation of colour differences as vector distances in those spaces. Industrial practice with small colour differences has shown non-uniform effects with calculated values in different ranges and different directions in those spaces. Moreover, a change of external observing conditions may change the perceived magnitude of the colour difference in a sample pair. The work documented in CIE 101-1993 (CIE, 1993) describes a number of external parameters of a visual task that affect the correlation of visual magnitude judgements of colour differences with their colorimetric measures. The outcome of those studies was the definition of reference conditions of a visual task to which a colour-difference formula should be adapted. Experimental data sets must now be chosen to meet reference conditions or to clearly define quantified deviations from them.

The CIEDE2000 total colour difference formula corrects for the non-uniformity of the CIELAB colour space for small colour differences under reference conditions. Improvements to the calculation of total colour difference for industrial colour difference evaluation are made through corrections for the effects of lightness dependence, chroma dependence, hue dependence and hue-chroma interaction on perceived colour difference. The scaling along the a* axis is modified to correct for a non-uniformity observed with grey colours. The resulting recommendation is as follows (CIE, 2001a)[XII]:

$$\Delta E_{00} = \left[\left(\frac{\Delta L'}{k_L S_L}\right)^2 + \left(\frac{\Delta C'}{k_C S_C}\right)^2 + \left(\frac{\Delta H'}{k_H S_H}\right)^2 + R_T\left(\frac{\Delta C'}{k_C S_C}\right)\left(\frac{\Delta H'}{k_H S_H}\right)\right]^{1/2} \quad (8.36)$$

A localized modification of the scaling along the a* (red-green opponent) axis is made to improve agreement with visual colour-difference perception for neutral colours. The modification increases the magnitudes of *a'* values compared to *a** values for colours at low chroma. The transformation is as follows:

$$L' = L^*$$
$$a' = a^*(1+G) \quad (8.37)$$
$$b' = b^*$$

$$G = 0{,}5\left(1 - \sqrt{\frac{\overline{C}^{*7}_{ab}}{\overline{C}^{*7}_{ab} + 25^7}}\right) \quad (8.38)$$

The transformed *L'*, *a'*, *b'* values are used in calculation of hue angle, chroma and lightness, chroma and hue differences and these quantities are designated by a prime mark in the equations.

[XII] We follow here the terminology used in CIE 142-2001 (CIE, 2001a), and omit the subscript "10", although CIEDE2000 is recommended for sample size larger than 4 degrees and thus the 10 degree observer has to be used.

Weighting functions, S_L, S_C, S_H adjust the total colour-difference for variation in perceived magnitude with variation in the location of the colour-difference pair in *L'*, *a'*, *b'* coordinates.

$$S_L = 1 + \frac{0{,}015\left(\bar{L}'-50\right)^2}{\sqrt{20+\left(\bar{L}'-50\right)^2}} \qquad (8.39)$$

$$S_C = 1 + 0{,}045\bar{C}' \qquad (8.40)$$

$$S_H = 1 + 0{,}015\bar{C}'T \qquad (8.41)$$

$$T = 1 - 0{,}17\cos\left(\bar{h}'-30\right) + 0{,}24\cos\left(2\bar{h}'\right) + 0{,}32\cos\left(3\bar{h}'+6\right) - 0{,}20\cos\left(4\bar{h}'-63\right) \qquad (8.42)$$

Visual colour-difference perception data show an interaction between chroma difference and hue difference in the blue region that is observed as a tilt of the major axis of a colour-difference ellipsoid from the direction of constant hue angle. To account for this effect, a rotation function is applied to weighted hue and chroma differences.

$$R_T = -\sin(2\Delta\Theta)R_C \qquad (8.43)$$

$$\Delta\Theta = 30\exp\left\{-\left[\left(\bar{h}'-275\right)/25\right]^2\right\} \qquad (8.44)$$

$$R_C = 2\sqrt{\frac{\bar{C}'^7}{\bar{C}'^7+25^7}} \qquad (8.45)$$

Mean hue angle and $\Delta\Theta$ values are in degree units.

Note 1: All quantities with a super-position bar indicate the mean of the values for each of the samples of a colour-difference pair.

Note 2: The parametric factors, k_L, k_C, k_H are correction terms for variation in experimental conditions. Under reference conditions they are all set at 1. For other choices see (CIE, 1993). The reference conditions are:

Illumination:	source simulating the spectral relative irradiance of CIE standard illuminant D65.
Illuminance:	1000 lx.
Observer:	normal colour vision.
Background field:	uniform, neutral grey with $L^* = 50$.
Viewing mode:	object.
Sample size:	greater than 4 degrees subtended visual angle.
Sample separation:	minimum sample separation achieved by placing the sample pair in direct edge contact.
Sample colour-difference magnitude:	0 to 5 CIELAB units.
Sample structure:	homogeneous colour without visually apparent pattern or non-uniformity.

Note 3: An alternative colour difference formula (CMC formula) not developed by the CIE, but used by some ISO committees, is briefly described in Appendix A.5 for reference purposes.

Note 4: A further colour difference formula based on a modified colour space derived from CIELAB is the DIN 99 formula (DIN, 2003), see Appendix A 6.

9 RECOMMENDATIONS CONCERNING MISCELLANEOUS COLORIMETRIC PRACTICES AND FORMULAE

9.1 Dominant wavelength and purity[19]

When it is desired to express chromaticity in terms of dominant (or complementary) wavelength and purity, it is recommended that the evaluations be carried out as follows.

9.1.1 Dominant wavelength (of a colour stimulus), λ_d

Wavelength of the monochromatic stimulus that, when additively mixed in suitable proportions with the specified achromatic stimulus, matches the colour stimulus considered.

Note: For stimuli whose chromaticities lie between those of the specified achromatic stimulus and the two ends of the spectrum, complementary wavelength is used instead of dominant wavelength.

9.1.2 Complementary wavelength (of a colour stimulus), λ_c

Wavelength of the monochromatic stimulus that, when additively mixed in suitable proportions with the colour stimulus considered, matches the specified achromatic stimulus.

Note: See note to dominant wavelength.

9.1.3 Colorimetric purity, p_c

Quantity, p_c, defined by the relation

$$p_c = L_d/(L_d + L_n) \tag{9.1}$$

where L_d and L_n are, respectively, the luminances of the monochromatic stimulus and of the specified achromatic stimulus that match the colour stimulus considered in an additive mixture.

Note 1: In the case of stimuli characterized by complementary wavelength, suitable mixtures of light from the two ends of the spectrum are used instead of the monochromatic stimulus, and colorimetric purity should then be calculated using the equation given in Note 2.

Note 2: In the CIE 1931 standard colorimetric system, colorimetric purity is related to excitation purity (see 9.1.4), p_e, by the equation

$$p_c = p_e y_d / y \tag{9.2}$$

where y_d and y are the y-chromaticity coordinates, respectively, of the monochromatic stimulus and the colour stimulus considered.

Note 3: In the CIE 1964 standard colorimetric system, the colorimetric purity, $p_{c,10}$ is defined by the relation given in Note 2, but using $p_{e,10}$, $y_{d,10}$, and y_{10} instead of p_e, y_d, and y.

9.1.4 Excitation purity, p_e

Quantity, p_e, defined by the ratio *NC*/*ND* of two collinear distances on the chromaticity diagram of the CIE 1931 or 1964 standard colorimetric system, the first distance being that between the point *C* representing the colour stimulus considered and the point *N* representing the specified achromatic stimulus; the second distance being that between the point *N* and the point *D* on the spectrum locus at the dominant wavelength of the colour stimulus considered. The definition leads to the following expressions:

$$p_e = \frac{y - y_n}{y_d - y_n} \quad \text{or} \quad p_e = \frac{x - x_n}{x_d - x_n} \tag{9.3}$$

where (x, y), (x_n, y_n), (x_d, y_d) are the x, y chromaticity coordinates of the points *C*, *N* and *D*, respectively.

Note 1: For colour stimuli for which no dominant wavelength exists, see Note 1 under colorimetric purity (Section 9.1.3).

Note 2: The formulae in x and y are equivalent but the use of that which has the greater value in the numerator results in greater precision.

Note 3: Excitation purity is related to colorimetric purity by the equation:

$$p_e = p_c\, y/y_d,\ \text{or}\ p_{e,10} = p_{c,10}\, y_{10}/y_{d,10}$$

9.2 Special metamerism indices[20]

Two specimens having identical tristimulus values for a given reference illuminant and reference observer are metameric if their spectral radiance distributions differ within the visible spectrum. The procedures concerned with a special metamerism index for a change from a reference illuminant to a test illuminant of different spectral composition, or that for a change from a reference observer to a test observer of different colour-matching functions are called the determination of special metamerism indices.

A measure of the metamerism for the two specimens is the colour difference between the two metameric specimens caused by substituting an illuminant, "special metamerism index: change in illuminant", and caused by substituting an observer, "special metamerism index: change in observer". The colour difference is evaluated using a CIE colour difference formula and it must be clearly stated which formula has been used.

It is recommended that for two specimens whose corresponding tristimulus values ($X_1 = X_2$, $Y_1 = Y_2$, $Z_1 = Z_2$) are identical with respect to a reference illuminant and observer, the metamerism index, M, be set equal to the colour difference ΔE^*_{ab} between the two specimens computed for the test illuminant or for the test observer.

9.2.1 Special metamerism index: change in illuminant

This procedure defines a special metamerism index M_{ilm} for a change from a reference illuminant to a test illuminant of different spectral composition.

Note: The metamerism index M_{ilm} is not suitable for determining the resultant colour shift or specifying the colour constancy of a single object colour when the illuminant is changed.

9.2.1.1 Tristimulus values under reference illuminant

For a pair of metameric object colours, their tristimulus values $X_{r,i}$, $Y_{r,i}$, $Z_{r,i}$ (i = 1, 2) under a reference illuminant are computed by the usual CIE recommendation (see Section 7) as

$$X_{m,i} = k \sum_\lambda \rho_i(\lambda)\, S(\lambda)\, \bar{x}(\lambda)\, \Delta\lambda$$

$$Y_{m,i} = k \sum_\lambda \rho_i(\lambda)\, S(\lambda)\, \bar{y}(\lambda)\, \Delta\lambda \tag{9.4}$$

$$Z_{m,i} = k \sum_\lambda \rho_i(\lambda)\, S(\lambda)\, \bar{z}(\lambda)\, \Delta\lambda$$

for m=r (reference illuminant), where $\rho_i(\lambda)$ is the spectral reflectance of the metameric pair (i = 1, 2), $S(\lambda)$ is the spectral power distribution of the reference illuminant, and $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, $\bar{z}(\lambda)$ are the colour-matching functions of either the CIE 1931 or 1964 standard colorimetric observers. It should be stated which observer has been used, and

$$k = \frac{100}{\sum_\lambda S(\lambda)\cdot\bar{y}(\lambda)\cdot\Delta\lambda} \tag{9.5}$$

The preferred reference illuminant is CIE standard illuminant D65. If another illuminant is used as reference, this should be noted.

For the set of the tristimulus values $X_{r,i}$, $Y_{r,i}$, $Z_{r,i}$ (i = 1,2), the following relation holds by definition.

$$X_{r,1} = X_{r,2},\quad Y_{r,1} = Y_{r,2},\quad Z_{r,1} = Z_{r,2} \tag{9.6}$$

If Equ. 9.6 fails to hold exactly, a suitable account should be taken of this failure. The nature of such account should be completely specified and the size of the failure recorded. See Note 1 after 9.2.2.3.

9.2.1.2 Tristimulus values under test illuminant

For the same pair of metameric object colours, their tristimulus values $X_{t,i}, Y_{t,i}, Z_{t,i}$ (i = 1, 2) under test illuminant are computed from Equ. 9.4 with m=t, by inserting the spectral power distribution of the test illuminant into $S(\lambda)$.

Suitable test illuminants include CIE standard illuminant A and the FL- and HP-illuminants defined in Table T.6 and T.7.

The FL-illuminants represent typical fluorescent lamps[21]. Colorimetric data for these illuminants are given in Table T.8. For the colour rendering index calculation the method as described in CIE 13.3 (CIE, 1995b) was used. The most appropriate choice of test illuminant depends upon application, but where only a few typical FL-illuminants are to be selected, FL2, FL7 and FL11 should take priority.

The HP-illuminants are typical high-pressure lamp spectra used at the time of publishing this report[22]. It may be useful to determine the metamerism index with respect to several test illuminants. The specific test illuminant used must be identified as a subscript to M, e.g. $M_{\mathrm{A,ilm}}$ or $M_{\mathrm{FL11,ilm}}$, etc.

9.2.1.3 Colour difference and metamerism index

The colour difference ΔE^*_{ab} is computed between the tristimulus values $X_{t,1}$, $Y_{t,1}$, $Z_{t,1}$ of object colour 1, and $X_{t,2}$, $Y_{t,2}$, $Z_{t,2}$ of object colour 2. Then the metamerism index M_{ilm} is defined as

$$M_{\mathrm{ilm}} = \Delta E^*_{\mathrm{ab}} \qquad (9.7)$$

If colour difference formulas other than CIELAB are used, this should be noted.

9.2.2 Special metamerism index: change in observer

The CIE 1931 and 1964 standard colorimetric observers represent the colour vision properties of the average population reasonably well. Nevertheless it is well known that individual deviations in the colour-matching functions occur among colour normal observers.

The special metamerism index: change in observer (CIE, 1989) was introduced to describe the average degree of mismatch found among metameric colours if the colour-matching functions of one of the standard colorimetric observers are changed to those of a standard deviate observer of normal colour vision.

9.2.2.1 Tristimulus values for standard colorimetric observers

For a pair of metameric object colours, their tristimulus values $X_{r,i}, Y_{r,i}, Z_{r,i}$ (i = 1, 2) for the standard colorimetric (reference) observers are computed from Equ. 9.4, with m=r, by use of either the CIE 1931 or 1964 colour-matching functions.

If the two metameric object colours fail to be a precise match with respect to the standard colorimetric observer, a suitable account should be taken of this failure. The nature of such account should be completely specified and the size of the failure recorded. See Note 1 after 9.2.2.3.

9.2.2.2 Tristimulus values for standard deviate observer

For the same pair of metameric object colours, their tristimulus values $X_{t,i}, Y_{t,i}, Z_{t,i}$ (i = 1, 2) for the standard deviate (test) observer are computed from Equ. 9.4, with m=t, by inserting the colour-matching functions $\bar{x}_d(\lambda)$, $\bar{y}_d(\lambda)$, $\bar{z}_d(\lambda)$ of the standard deviate observer. The colour-matching functions $\bar{x}_d(\lambda)$, $\bar{y}_d(\lambda)$, $\bar{z}_d(\lambda)$ are given by

$$\bar{x}_d(\lambda) = \bar{x}(\lambda) + \Delta\bar{x}(\lambda)$$

$$\bar{y}_d(\lambda) = \bar{y}(\lambda) + \Delta\bar{y}(\lambda) \qquad (9.8)$$

$$\bar{z}_d(\lambda) = \bar{z}(\lambda) + \Delta\bar{z}(\lambda)$$

where $\Delta\bar{x}(\lambda)$, $\Delta\bar{y}(\lambda)$, $\Delta\bar{z}(\lambda)$ are the so-called first deviation functions defined in Table 1.1 of (CIE, 1989) and are reproduced in Table T.10.

9.2.2.3 Colour difference and metamerism index

The colour difference ΔE^*_{ab} is computed between the tristimulus values $X_{t,1}$, $Y_{t,1}$, $Z_{t,1}$ of object colour 1, and $X_{t,2}$, $Y_{t,2}$, $Z_{t,2}$ of object colour 2. Then the metamerism index M_{obs} is defined as

$$M_{obs} = \Delta E^*_{ab} \qquad (9.9)$$

If a colour difference formula other than CIELAB is used, this should be included in the parenthesis as e.g. $M_{obs}(u^*v^*)$.

Notes on special metamerism indices

Note 1: When the samples are not exactly metameric, that is $X_{r,1} \neq X_{r,2}$, $Y_{r,1} \neq Y_{r,2}$, $Z_{r,1} \neq Z_{r,2}$, then the tristimulus values $X_{t,2}$, $Y_{t,2}$, $Z_{t,2}$ are adjusted by the multiplicative method as follows.

$$X'_{t,2} = X_{t,2}(X_{r,1} / X_{r,2}),\ Y'_{t,2} = Y_{t,2}(Y_{r,1} / Y_{r,2}),\ Z'_{t,2} = Z_{t,2}(Z_{r,1} / Z_{r,2}) \qquad (9.10)$$

Note 2: Each colour normal observer shows a colour difference ΔE^*_i for a sample pair metameric with respect to a reference observer and an irradiating illuminant. About 95 % of ΔE^*s for colour normal observers are usually found to be within 2 ΔM_{obs} (a^*b^* or u^*v^*).

Note 3: Further characterization of the observer metamerism can be done by calculating the range of colour mismatch, see (CIE, 1989).

9.3 Assessment of the quality of a daylight simulator for colorimetry[23]

The quality of simulators of CIE daylight illuminants D50, D55, D65 and D75 can be assessed by calculating the special metamerism index for change in illuminant, employing specified samples that are metameric matches for the CIE illuminants D50, D55, D65 and D75, respectively, and the CIE 1964 standard colorimetric observer. The purpose of this assessment is to quantify the suitability of a test source as a practical reproduction of CIE standard illuminant D50, D55, D65, or D75 for colorimetric tasks[24].

The basis for the assessment is the special metamerism index: change in illuminant (see Section 9.2.1), employing specified samples that are metameric matches for the standard daylight illuminant and the CIE 1964 standard colorimetric observer. The method quantifies the mismatch resulting when samples that are a match under the standard daylight illuminant are viewed under the illumination of the test source, the CIE 1964 standard colorimetric observer being used throughout.

A visible range metamerism index is employed to evaluate the colorimetric suitability of the test source for the visible wavelength range. Tables of spectral reflection radiance factor define the metameric samples.

An ultraviolet range metamerism index is employed with a second set of metameric samples to evaluate the suitability of the test source in relation to ultraviolet-excited luminescent colours. The metameric sample pairs for this assessment are comprised of a luminescent and a non-luminescent sample, which are spectrally identical matches for the standard daylight illuminant. The non-luminescent sample in each metameric pair is specified by values of spectral reflection radiance factor for each standard daylight illuminant (D50, D55, D65, and D75).

The luminescent sample in each metameric pair is specified by values of spectral reflection radiance factor, relative spectral distribution of radiance emitted by fluorescence, and spectral external radiant efficiency.

The ultraviolet range metamerism index quantifies the failure of the spectrally identical match between the luminescent and the non-luminescent sample pairs resulting from changing the illuminating source from a standard daylight illuminant to a test source.

Detailed description of the test method, including the tables needed for calculating the quality index, and a computer program of the calculation method on disk, are published in CIE 51.2-1999 (CIE, 1999c), see also (CIE, 2001b).

9.4 The evaluation of whiteness[25]

To promote uniformity of practice in the evaluation of whiteness of surface colours, it is recommended that the formulae for whiteness, W or W_{10}, and for tint, T_w or $T_{w,10}$, given below, be used for comparisons of the whiteness of samples evaluated for CIE standard illuminant D65. The application of the formulae is restricted to samples that are called "white" commercially, that do not differ much in colour and fluorescence, and that are measured on the same instrument at nearly the same time. Within these restrictions, the formulae provide relative, but not absolute, evaluations of whiteness, that are adequate for commercial use, when employing measuring instruments having suitable modern and commercially available facilities.

$$\begin{aligned} W &= Y + 800(x_n - x) + 1700(y_n - y) \\ W_{10} &= Y_{10} + 800(x_{n,10} - x_{10}) + 1700(y_{n,10} - y_{10}) \\ T_w &= 1000(x_n - x) - 650(y_n - y) \\ T_{w,10} &= 900(x_{n,10} - x_{10}) - 650(y_{n,10} - y_{10}) \end{aligned} \tag{9.11}$$

where Y is the Y-tristimulus value of the sample, x and y are the x, y chromaticity coordinates of the sample, and x_n, y_n are the chromaticity coordinates of the perfect diffuser, all for the CIE 1931 standard colorimetric observer; Y_{10}, x_{10} , y_{10}, $x_{n,10}$ and $y_{n,10}$ are similar values for the CIE 1964 standard colorimetric observer.

Note 1: The higher the value of W or W_{10}, the greater is the indicated whiteness. The more positive the value of T_w or $T_{w,10}$, the greener the tint; the more negative the value of T_w or $T_{w,10}$, the redder the tint. For the perfect diffuser W and W_{10} are equal to 100, and T_w and $T_{w,10}$ are equal to zero.

Note 2: Linear whiteness formulae are applicable only within a restricted volume of the colour solid. These formulae may be used only for samples whose values of W or W_{10} and T_w or $T_{w,10}$ lie within the following limits:

W or W_{10}	greater than 40 and less than $5Y - 280$, or $5Y_{10} - 280$;
T_w or $T_{w,10}$	greater than -4 and less than +2.

Note 3: The tint formulae are based on the empirical results that lines of equal tint run approximately parallel to lines of dominant wavelength 466 nm in the x, y and x_{10}, y_{10} chromaticity diagrams.

Note 4: Equal differences in W or W_{10} do not always represent equal perceptual differences in whiteness, nor do equal differences in T_w or $T_{w,10}$ always represent equal perceptual differences in greenishness or reddishness of whites. Measures of whiteness and tint that correlate uniformly with these perceptual attributes would require more complicated formulae, which is beyond present knowledge.

9.5 Calculation of correlated colour temperature[26]

The concept of correlated colour temperature used to be based on visual observations. Recent investigations have shown (see Borbély et al., 2001) that no metrological definition can be based on such perceptual investigations. Therefore a new definition has been proposed. The definition agrees with the previously recommended calculation method and thus does not cause any changes to calculated values.

correlated colour temperature (T_{cp})

temperature of a Planckian radiator having the chromaticity nearest the chromaticity associated with the given spectral distribution on a diagram where the (CIE 1931 standard observer based) u', $2/3v'$ coordinates of the Planckian locus[XIII] and the test stimulus are depicted

Note 1: The concept of correlated colour temperature should not be used if the chromaticity of the test source differs more than $\Delta C = [(u'_t-u'_P)^2+\frac{4}{9}\cdot(v'_t-v'_P)^2]^{1/2} = 5\cdot10^{-2}$ from the Planckian radiator, where u'_t,v'_t refer to the test source, u'_P,v'_P to the Planckian radiator.

Note 2: Correlated colour temperature can be calculated by a simple minimum search computer program that searches for that Planckian temperature that provides the smallest chromaticity difference between the test chromaticity and the Planckian locus, or e.g. by a method recommended by Robertson (1968).

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[XIII] In calculating the chromaticity coordinates of the Planckian radiator the c_2 value according to ITC-90 has to be used (c_2 = 1,4388) in Planck's equation for standard air, but assuming n=1. See Appendix E.

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11. TABLES

This Section of the report contains tables of abridged and truncated colorimetric data. The full tables – recommended for general use – are found in the CIE standards on colorimetry[XIV]. The tables presented here are intended for use, only when the highest precision is not required. Users should check before using these tables whether the level of precision meets their needs.

The illuminant tables in this section are provided for the range between 300 nm and 780 nm. The tables of other colorimetric functions are provided for the range between 380 nm and 780 nm. All tables are at 5 nm intervals.

11.1 Table T.1. Relative spectral power distributions of CIE illuminants

Relative spectral power distributions [$S(\lambda)$] of CIE standard illuminants A and D65, as well as CIE illuminants C, D50, D55 and D75 for wavelengths λ = 300 nm to 780 nm at 5 nm intervals[XV]

λ, nm	Standard Illuminant A	Standard Illuminant D65	Illuminant C	Illuminant D50	Illuminant D55	Illuminant D75
300	0,930 483	0,034 100 0	0,00	0,019	0,024	0,043
305	1,128 21	1,664 30	0,00	1,035	1,048	2,588
310	1,357 69	3,294 50	0,00	2,051	2,072	5,133
315	1,622 19	11,765 2	0,00	4,914	6,648	17,470
320	1,925 08	20,236 0	0,01	7,778	11,224	29,808
325	2,269 80	28,644 7	0,20	11,263	15,936	42,369
330	2,659 81	37,053 5	0,40	14,748	20,647	54,930
335	3,098 61	38,501 1	1,55	16,348	22,266	56,095
340	3,589 68	39,948 8	2,70	17,948	23,885	57,259
345	4,136 48	42,430 2	4,85	19,479	25,851	60,000
350	4,742 38	44,911 7	7,00	21,010	27,817	62,740
355	5,410 70	45,775 0	9,95	22,476	29,219	62,861
360	6,144 62	46,638 3	12,90	23,942	30,621	62,982
365	6,947 20	49,363 7	17,20	25,451	32,464	66,647
370	7,821 35	52,089 1	21,40	26,961	34,308	70,312
375	8,769 80	51,032 3	27,50	25,724	33,446	68,507
380	9,795 10	49,975 5	33,00	24,488	32,584	66,703
385	10,899 6	52,311 8	39,92	27,179	35,335	68,333
390	12,085 3	54,648 2	47,40	29,871	38,087	69,963
395	13,354 3	68,701 5	55,17	39,589	49,518	85,946
400	14,708 0	82,754 9	63,30	49,308	60,949	101,929
405	16,148 0	87,120 4	71,81	52,910	64,751	106,911
410	17,675 3	91,486 0	80,60	56,513	68,554	111,894
415	19,290 7	92,458 9	89,53	58,273	70,065	112,346
420	20,995 0	93,431 8	98,10	60,034	71,577	112,798

[XIV] These tables can also be found on the accompanying CD-ROM.

[XV] Standard illuminant A and standard illuminant D65 data are every fifth value from the CIE Standard. The illuminant C data are unchanged from CIE 15.2-1986. The D50, D55 and D75 data are calculated according to the present version of CIE 15.

λ, nm	Standard Illuminant A	Standard Illuminant D65	Illuminant C	Illuminant D50	Illuminant D55	Illuminant D75
425	22,788 3	90,057 0	105,80	58,926	69,746	107,945
430	24,670 9	86,682 3	112,40	57,818	67,914	103,092
435	26,642 5	95,773 6	117,75	66,321	76,760	112,145
440	28,702 7	104,865	121,50	74,825	85,605	121,198
445	30,850 8	110,936	123,45	81,036	91,799	127,104
450	33,085 9	117,008	124,00	87,247	97,993	133,010
455	35,406 8	117,410	123,60	88,930	99,228	132,682
460	37,812 1	117,812	123,10	90,612	100,463	132,355
465	40,300 2	116,336	123,30	90,990	100,188	129,838
470	42,869 3	114,861	123,80	91,368	99,913	127,322
475	45,517 4	115,392	124,09	93,238	101,326	127,061
480	48,242 3	115,923	123,90	95,109	102,739	126,800
485	51,041 8	112,367	122,92	93,536	100,409	122,291
490	53,913 2	108,811	120,70	91,963	98,078	117,783
495	56,853 9	109,082	116,90	93,843	99,379	117,186
500	59,861 1	109,354	112,10	95,724	100,680	116,589
505	62,932 0	108,578	106,98	96,169	100,688	115,146
510	66,063 5	107,802	102,30	96,613	100,695	113,702
515	69,252 5	106,296	98,81	96,871	100,341	111,181
520	72,495 9	104,790	96,90	97,129	99,987	108,659
525	75,790 3	106,239	96,78	99,614	102,098	109,552
530	79,132 6	107,689	98,00	102,099	104,210	110,445
535	82,519 3	106,047	99,94	101,427	103,156	108,367
540	85,947 0	104,405	102,10	100,755	102,102	106,289
545	89,412 4	104,225	103,95	101,536	102,535	105,596
550	92,912 0	104,046	105,20	102,317	102,968	104,904
555	96,442 3	102,023	105,67	101,159	101,484	102,452
560	100,000	100,000	105,30	100,000	100,000	100,000
565	103,582	98,167 1	104,11	98,868	98,608	97,808
570	107,184	96,334 2	102,30	97,735	97,216	95,616
575	110,803	96,061 1	100,15	98,327	97,482	94,914
580	114,436	95,788 0	97,80	98,918	97,749	94,213
585	118,080	92,236 8	95,43	96,208	94,590	90,605
590	121,731	88,685 6	93,20	93,499	91,432	86,997
595	125,386	89,345 9	91,22	95,593	92,926	87,112
600	129,043	90,006 2	89,70	97,688	94,419	87,227
605	132,697	89,802 6	88,83	98,478	94,780	86,684
610	136,346	89,599 1	88,40	99,269	95,140	86,140
615	139,988	88,648 9	88,19	99,155	94,680	84,861
620	143,618	87,698 7	88,10	99,042	94,220	83,581
625	147,235	85,493 6	88,06	97,382	92,334	81,164
630	150,836	83,288 6	88,00	95,722	90,448	78,747

λ, nm	Standard Illuminant A	Standard Illuminant D65	Illuminant C	Illuminant D50	Illuminant D55	Illuminant D75
635	154,418	83,493 9	87,86	97,290	91,389	78,587
640	157,979	83,699 2	87,80	98,857	92,330	78,428
645	161,516	81,863 0	87,99	97,262	90,592	76,614
650	165,028	80,026 8	88,20	95,667	88,854	74,801
655	168,510	80,120 7	88,20	96,929	89,586	74,562
660	171,963	80,214 6	87,90	98,190	90,317	74,324
665	175,383	81,246 2	87,22	100,597	92,133	74,873
670	178,769	82,277 8	86,30	103,003	93,950	75,422
675	182,118	80,281 0	85,30	101,068	91,953	73,499
680	185,429	78,284 2	84,00	99,133	89,956	71,576
685	188,701	74,002 7	82,21	93,257	84,817	67,714
690	191,931	69,721 3	80,20	87,381	79,677	63,852
695	195,118	70,665 2	78,24	89,492	81,258	64,464
700	198,261	71,609 1	76,30	91,604	82,840	65,076
705	201,359	72,979 0	74,36	92,246	83,842	66,573
710	204,409	74,349 0	72,40	92,889	84,844	68,070
715	207,411	67,976 5	70,40	84,872	77,539	62,256
720	210,365	61,604 0	68,30	76,854	70,235	56,443
725	213,268	65,744 8	66,30	81,683	74,768	60,343
730	216,120	69,885 6	64,40	86,511	79,301	64,242
735	218,920	72,486 3	62,80	89,546	82,147	66,697
740	221,667	75,087 0	61,50	92,580	84,993	69,151
745	224,361	69,339 8	60,20	85,405	78,437	63,890
750	227,000	63,592 7	59,20	78,230	71,880	58,629
755	229,585	55,005 4	58,50	67,961	62,337	50,623
760	232,115	46,418 2	58,10	57,692	52,793	42,617
765	234,589	56,611 8	58,00	70,307	64,360	51,985
770	237,008	66,805 4	58,20	82,923	75,927	61,352
775	239,370	65,094 1	58,50	80,599	73,872	59,838
780	241,675	63,382 8	59,10	78,274	71,818	58,324

11.2 Table T.2. Components $S_0(\lambda)$, $S_1(\lambda)$, $S_2(\lambda)$

Components $S_0(\lambda)$, $S_1(\lambda)$, $S_2(\lambda)$ of daylight used in the calculation of relative spectral power distributions of CIE daylight illuminants of different correlated colour temperatures, for wavelengths λ = 300 nm to 830 nm at 5 nm intervals.

λ, nm	$S_0(\lambda)$	$S_1(\lambda)$	$S_2(\lambda)$
300	0,04	0,02	0,00
305	3,02	2,26	1,00
310	6,00	4,50	2,00
315	17,80	13,45	3,00
320	29,60	22,40	4,00
325	42,45	32,20	6,25
330	55,30	42,00	8,50
335	56,30	41,30	8,15
340	57,30	40,60	7,80
345	59,55	41,10	7,25
350	61,80	41,60	6,70
355	61,65	39,80	6,00
360	61,50	38,00	5,30
365	65,15	40,20	5,70
370	68,80	42,40	6,10
375	66,10	40,45	4,55
380	63,40	38,50	3,00
385	64,60	36,75	2,10
390	65,80	35,00	1,20
395	80,30	39,20	0,05
400	94,80	43,40	-1,10
405	99,80	44,85	-0,80
410	104,80	46,30	-0,50
415	105,35	45,10	-0,60
420	105,90	43,90	-0,70
425	101,35	40,50	-0,95
430	96,80	37,10	-1,20
435	105,35	36,90	-1,90
440	113,90	36,70	-2,60
445	119,75	36,30	-2,75
450	125,60	35,90	-2,90
455	125,55	34,25	-2,85
460	125,50	32,60	-2,80
465	123,40	30,25	-2,70
470	121,30	27,90	-2,60
475	121,30	26,10	-2,60
480	121,30	24,30	-2,60
485	117,40	22,20	-2,20
490	113,50	20,10	-1,80
495	113,30	18,15	-1,65
500	113,10	16,20	-1,50
505	111,95	14,70	-1,40
510	110,80	13,20	-1,30
515	108,65	10,90	-1,25
520	106,50	8,60	-1,20
525	107,65	7,35	-1,10
530	108,80	6,10	-1,00
535	107,05	5,15	-0,75
540	105,30	4,20	-0,50
545	104,85	3,05	-0,40
550	104,40	1,90	-0,30
555	102,20	0,95	-0,15
560	100,00	0,00	0,00
565	98,00	-0,80	0,10
570	96,00	-1,60	0,20
575	95,55	-2,55	0,35
580	95,10	-3,50	0,50
585	92,10	-3,50	1,30
590	89,10	-3,50	2,10
595	89,80	-4,65	2,65
600	90,50	-5,80	3,20
605	90,40	-6,50	3,65
610	90,30	-7,20	4,10
615	89,35	-7,90	4,40
620	88,40	-8,60	4,70
625	86,20	-9,05	4,90
630	84,00	-9,50	5,10
635	84,55	-10,20	5,90
640	85,10	-10,90	6,70
645	83,50	-10,80	7,00
650	81,90	-10,70	7,30
655	82,25	-11,35	7,95
660	82,60	-12,00	8,60
665	83,75	-13,00	9,20
670	84,90	-14,00	9,80
675	83,10	-13,80	10,00

λ, nm	$S_0(\lambda)$	$S_1(\lambda)$	$S_2(\lambda)$
680	81,30	-13,60	10,20
685	76,60	-12,80	9,25
690	71,90	-12,00	8,30
695	73,10	-12,65	8,95
700	74,30	-13,30	9,60
705	75,35	-13,10	9,05
710	76,40	-12,90	8,50
715	69,85	-11,75	7,75
720	63,30	-10,60	7,00
725	67,50	-11,10	7,30
730	71,70	-11,60	7,60
735	74,35	-11,90	7,80
740	77,00	-12,20	8,00
745	71,10	-11,20	7,35
750	65,20	-10,20	6,70
755	56,45	-9,00	5,95
760	47,70	-7,80	5,20

λ, nm	$S_0(\lambda)$	$S_1(\lambda)$	$S_2(\lambda)$
765	58,15	-9,50	6,30
770	68,60	-11,20	7,40
775	66,80	-10,80	7,10
780	65,00	-10,40	6,80
785	65,50	-10,50	6,90
790	66,00	-10,60	7,00
795	63,50	-10,15	6,70
800	61,00	-9,70	6,40
805	57,15	-9,00	5,95
810	53,30	-8,30	5,50
815	56,10	-8,80	5,80
820	58,90	-9,30	6,10
825	60,40	-9,55	6,30
830	61,90	-9,80	6,50

11.3 Table T.3. Tristimulus values, chromaticity coordinates of CIE illuminants

Tristimulus values, X, Y, Z, and chromaticity coordinates x, y, and u', v' for CIE standard illuminants A and D65, as well as for CIE illuminants C, D50, D55 and D75 computed for the 1931 and 1964 standard colorimetric observers.

1. For the CIE 1931 standard colorimetric observer as defined in Table T.4 and illuminants as defined in Table T.1 (5 nm intervals over the range 380 nm to 780 nm).

	Standard Illuminant A	Standard Illuminant D65	Illuminant C	Illuminant D50	Illuminant D55	Illuminant D75
X	109,85	95,04	98,07	96,42	95,68	94,97
Y	100,00	100,00	100,00	100,00	100,00	100,00
Z	35,58	108,88	118,22	82,51	92,14	122,61
x	0,447 58	0,312 72	0,310 06	0,345 67	0,332 43	0,299 03
y	0,407 45	0,329 03	0,316 16	0,358 51	0,347 44	0,314 88
u'	0,255 97	0,197 83	0,200 89	0,209 16	0,204 43	0,193 53
v'	0,524 29	0,468 34	0,460 89	0,488 08	0,480 75	0,458 53

2. For the CIE 1964 standard colorimetric observer as defined in Table T.5 and illuminants as defined in Table T.1 (5 nm intervals over the range 380 nm to 780 nm).

	Standard Illuminant A	Standard Illuminant D65	Illuminant C	Illuminant D50	Illuminant D55	Illuminant D75
X_{10}	111,14	94,81	97,29	96,72	95,80	94,42
Y_{10}	100,00	100,00	100,00	100,00	100,00	100,00
Z_{10}	35,20	107,32	116,14	81,43	90,93	120,64
x_{10}	0,451 17	0,313 81	0,310 39	0,347 73	0,334 12	0,299 68
y_{10}	0,405 94	0,330 98	0,319 05	0,359 52	0,348 77	0,317 40
u'_{10}	0,258 96	0,197 86	0,200 00	0,210 15	0,205 07	0,193 05
v'_{10}	0,524 25	0,469 54	0,462 55	0,488 86	0,481 65	0,460 04

11.4 Table T.4. CIE 1931 standard colorimetric observer

Truncated set of colour-matching functions $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, $\bar{z}(\lambda)$ and corresponding chromaticity coordinates $x(\lambda)$, $y(\lambda)$ for wavelengths λ = 380 nm to 780 nm at 5 nm intervals, rounded to 6 and 5 decimal places respectively.[XVI]

λ, nm	$\bar{x}(\lambda)$	$\bar{y}(\lambda)$	$\bar{z}(\lambda)$	$x(\lambda)$	$y(\lambda)$
380	0,001 368	0,000 039	0,006 450	0,174 11	0,004 96
385	0,002 236	0,000 064	0,010 550	0,174 01	0,004 98
390	0,004 243	0,000 120	0,020 050	0,173 80	0,004 92
395	0,007 650	0,000 217	0,036 210	0,173 56	0,004 92
400	0,014 310	0,000 396	0,067 850	0,173 34	0,004 80
405	0,023 190	0,000 640	0,110 200	0,173 02	0,004 78
410	0,043 510	0,001 210	0,207 400	0,172 58	0,004 80
415	0,077 630	0,002 180	0,371 300	0,172 09	0,004 83
420	0,134 380	0,004 000	0,645 600	0,171 41	0,005 10
425	0,214 770	0,007 300	1,039 050	0,170 30	0,005 79
430	0,283 900	0,011 600	1,385 600	0,168 88	0,006 90
435	0,328 500	0,016 840	1,622 960	0,166 90	0,008 56
440	0,348 280	0,023 000	1,747 060	0,164 41	0,010 86
445	0,348 060	0,029 800	1,782 600	0,161 10	0,013 79
450	0,336 200	0,038 000	1,772 110	0,156 64	0,017 70
455	0,318 700	0,048 000	1,744 100	0,150 99	0,022 74
460	0,290 800	0,060 000	1,669 200	0,143 96	0,029 70
465	0,251 100	0,073 900	1,528 100	0,135 50	0,039 88
470	0,195 360	0,090 980	1,287 640	0,124 12	0,057 80
475	0,142 100	0,112 600	1,041 900	0,109 59	0,086 84
480	0,095 640	0,139 020	0,812 950	0,091 29	0,132 70
485	0,057 950	0,169 300	0,616 200	0,068 71	0,200 72
490	0,032 010	0,208 020	0,465 180	0,045 39	0,294 98
495	0,014 700	0,258 600	0,353 300	0,023 46	0,412 70
500	0,004 900	0,323 000	0,272 000	0,008 17	0,538 42
505	0,002 400	0,407 300	0,212 300	0,003 86	0,654 82
510	0,009 300	0,503 000	0,158 200	0,013 87	0,750 19
515	0,029 100	0,608 200	0,111 700	0,038 85	0,812 02
520	0,063 270	0,710 000	0,078 250	0,074 30	0,833 80
525	0,109 600	0,793 200	0,057 250	0,114 16	0,826 21
530	0,165 500	0,862 000	0,042 160	0,154 72	0,805 86
535	0,225 750	0,914 850	0,029 840	0,192 88	0,781 63
540	0,290 400	0,954 000	0,020 300	0,229 62	0,754 33
545	0,359 700	0,980 300	0,013 400	0,265 78	0,724 32
550	0,433 450	0,994 950	0,008 750	0,301 60	0,692 31

[XVI] Chromaticity coordinates have been calculated from the non-rounded values published in (CIE, 1986a).

λ, nm	$\bar{x}(\lambda)$	$\bar{y}(\lambda)$	$\bar{z}(\lambda)$	$x(\lambda)$	$y(\lambda)$
555	0,512 050	1,000 000	0,005 750	0,337 36	0,658 85
560	0,594 500	0,995 000	0,003 900	0,373 10	0,624 45
565	0,678 400	0,978 600	0,002 750	0,408 74	0,589 61
570	0,762 100	0,952 000	0,002 100	0,444 06	0,554 71
575	0,842 500	0,915 400	0,001 800	0,478 77	0,520 20
580	0,916 300	0,870 000	0,001 650	0,512 49	0,486 59
585	0,978 600	0,816 300	0,001 400	0,544 79	0,454 43
590	1,026 300	0,757 000	0,001 100	0,575 15	0,424 23
595	1,056 700	0,694 900	0,001 000	0,602 93	0,396 50
600	1,062 200	0,631 000	0,000 800	0,627 04	0,372 49
605	1,045 600	0,566 800	0,000 600	0,648 23	0,351 39
610	1,002 600	0,503 000	0,000 340	0,665 76	0,334 01
615	0,938 400	0,441 200	0,000 240	0,680 08	0,319 75
620	0,854 450	0,381 000	0,000 190	0,691 50	0,308 34
625	0,751 400	0,321 000	0,000 100	0,700 61	0,299 30
630	0,642 400	0,265 000	0,000 050	0,707 92	0,292 03
635	0,541 900	0,217 000	0,000 030	0,714 03	0,285 93
640	0,447 900	0,175 000	0,000 020	0,719 03	0,280 93
645	0,360 800	0,138 200	0,000 010	0,723 03	0,276 95
650	0,283 500	0,107 000	0,000 000	0,725 99	0,274 01
655	0,218 700	0,081 600	0,000 000	0,728 27	0,271 73
660	0,164 900	0,061 000	0,000 000	0,729 97	0,270 03
665	0,121 200	0,044 580	0,000 000	0,731 09	0,268 91
670	0,087 400	0,032 000	0,000 000	0,731 99	0,268 01
675	0,063 600	0,023 200	0,000 000	0,732 72	0,267 28
680	0,046 770	0,017 000	0,000 000	0,733 42	0,266 58
685	0,032 900	0,011 920	0,000 000	0,734 05	0,265 95
690	0,022 700	0,008 210	0,000 000	0,734 39	0,265 61
695	0,015 840	0,005 723	0,000 000	0,734 59	0,265 41
700	0,011 359	0,004 102	0,000 000	0,734 69	0,265 31
705	0,008 111	0,002 929	0,000 000	0,734 69	0,265 31
710	0,005 790	0,002 091	0,000 000	0,734 69	0,265 31
715	0,004 109	0,001 484	0,000 000	0,734 69	0,265 31
720	0,002 899	0,001 047	0,000 000	0,734 69	0,265 31
725	0,002 049	0,000 740	0,000 000	0,734 69	0,265 31
730	0,001 440	0,000 520	0,000 000	0,734 69	0,265 31
735	0,001 000	0,000 361	0,000 000	0,734 69	0,265 31
740	0,000 690	0,000 249	0,000 000	0,734 69	0,265 31
745	0,000 476	0,000 172	0,000 000	0,734 69	0,265 31
750	0,000 332	0,000 120	0,000 000	0,734 69	0,265 31
755	0,000 235	0,000 085	0,000 000	0,734 69	0,265 31
760	0,000 166	0,000 060	0,000 000	0,734 69	0,265 31

λ, nm	$\bar{x}(\lambda)$	$\bar{y}(\lambda)$	$\bar{z}(\lambda)$	$x(\lambda)$	$y(\lambda)$
765	0,000 117	0,000 042	0,000 000	0,734 69	0,265 31
770	0,000 083	0,000 030	0,000 000	0,734 69	0,265 31
775	0,000 059	0,000 021	0,000 000	0,734 69	0,265 31
780	0,000 042	0,000 015	0,000 000	0,734 69	0,265 31

Summation at 5 nm intervals:

$$\sum \bar{x}(\lambda) = 21{,}371524 \qquad \sum \bar{y}(\lambda) = 21{,}371327 \qquad \sum \bar{z}(\lambda) = 21{,}371\,540$$

11.5 Table T.5. CIE 1964 standard colorimetric observer

Truncated set of colour-matching functions $\bar{x}_{10}(\lambda)$, $\bar{y}_{10}(\lambda)$, $\bar{z}_{10}(\lambda)$ and chromaticity coordinates $x_{10}(\lambda)$, $y_{10}(\lambda)$ for wavelengths λ = 380 nm to 780 nm at 5 nm intervals, rounded to 6 decimal places and 5 decimal places respectively[XVII].

λ,nm	$\bar{x}_{10}(\lambda)$	$\bar{y}_{10}(\lambda)$	$\bar{z}_{10}(\lambda)$	$x_{10}(\lambda)$	$y_{10}(\lambda)$
380	0,000 160	0,000 017	0,000 705	0,181 33	0,019 69
385	0,000 662	0,000 072	0,002 928	0,180 91	0,019 54
390	0,002 362	0,000 253	0,010 482	0,180 31	0,019 35
395	0,007 242	0,000 769	0,032 344	0,179 47	0,019 04
400	0,019 110	0,002 004	0,086 011	0,178 39	0,018 71
405	0,043 400	0,004 509	0,197 120	0,177 12	0,018 40
410	0,084 736	0,008 756	0,389 366	0,175 49	0,018 13
415	0,140 638	0,014 456	0,656 760	0,173 23	0,017 81
420	0,204 492	0,021 391	0,972 542	0,170 63	0,017 85
425	0,264 737	0,029 497	1,282 500	0,167 90	0,018 71
430	0,314 679	0,038 676	1,553 480	0,165 03	0,020 28
435	0,357 719	0,049 602	1,798 500	0,162 17	0,022 49
440	0,383 734	0,062 077	1,967 280	0,159 02	0,025 73
445	0,386 726	0,074 704	2,027 300	0,155 39	0,030 02
450	0,370 702	0,089 456	1,994 800	0,151 00	0,036 44
455	0,342 957	0,106 256	1,900 700	0,145 94	0,045 22
460	0,302 273	0,128 201	1,745 370	0,138 92	0,058 92
465	0,254 085	0,152 761	1,554 900	0,129 52	0,077 87
470	0,195 618	0,185 190	1,317 560	0,115 18	0,109 04
475	0,132 349	0,219 940	1,030 200	0,095 73	0,159 09
480	0,080 507	0,253 589	0,772 125	0,072 78	0,229 24
485	0,041 072	0,297 665	0,570 060	0,045 19	0,327 54
490	0,016 172	0,339 133	0,415 254	0,020 99	0,440 11
495	0,005 132	0,395 379	0,302 356	0,007 30	0,562 52

[XVII] Chromaticity coordinates have been calculated from the non-rounded values published in (CIE, 1986a).

λ,nm	$\bar{x}_{10}(\lambda)$	$\bar{y}_{10}(\lambda)$	$\bar{z}_{10}(\lambda)$	$x_{10}(\lambda)$	$y_{10}(\lambda)$
500	0,003 816	0,460 777	0,218 502	0,005 59	0,674 54
505	0,015 444	0,531 360	0,159 249	0,021 87	0,752 58
510	0,037 465	0,606 741	0,112 044	0,049 54	0,802 30
515	0,071 358	0,685 660	0,082 248	0,085 02	0,816 98
520	0,117 749	0,761 757	0,060 709	0,125 24	0,810 19
525	0,172 953	0,823 330	0,043 050	0,166 41	0,792 17
530	0,236 491	0,875 211	0,030 451	0,207 06	0,766 28
535	0,304 213	0,923 810	0,020 584	0,243 64	0,739 87
540	0,376 772	0,961 988	0,013 676	0,278 59	0,711 30
545	0,451 584	0,982 200	0,007 918	0,313 23	0,681 28
550	0,529 826	0,991 761	0,003 988	0,347 30	0,650 09
555	0,616 053	0,999 110	0,001 091	0,381 16	0,618 16
560	0,705 224	0,997 340	0,000 000	0,414 21	0,585 79
565	0,793 832	0,982 380	0,000 000	0,446 92	0,553 08
570	0,878 655	0,955 552	0,000 000	0,479 04	0,520 96
575	0,951 162	0,915 175	0,000 000	0,509 64	0,490 36
580	1,014 160	0,868 934	0,000 000	0,538 56	0,461 44
585	1,074 300	0,825 623	0,000 000	0,565 44	0,434 56
590	1,118 520	0,777 405	0,000 000	0,589 96	0,410 04
595	1,134 300	0,720 353	0,000 000	0,611 60	0,388 40
600	1,123 990	0,658 341	0,000 000	0,630 63	0,369 37
605	1,089 100	0,593 878	0,000 000	0,647 13	0,352 87
610	1,030 480	0,527 963	0,000 000	0,661 22	0,338 78
615	0,950 740	0,461 834	0,000 000	0,673 06	0,326 94
620	0,856 297	0,398 057	0,000 000	0,682 66	0,317 34
625	0,754 930	0,339 554	0,000 000	0,689 76	0,310 24
630	0,647 467	0,283 493	0,000 000	0,695 48	0,304 52
635	0,535 110	0,228 254	0,000 000	0,700 99	0,299 01
640	0,431 567	0,179 828	0,000 000	0,705 87	0,294 13
645	0,343 690	0,140 211	0,000 000	0,710 25	0,289 75
650	0,268 329	0,107 633	0,000 000	0,713 71	0,286 29
655	0,204 300	0,081 187	0,000 000	0,715 62	0,284 38
660	0,152 568	0,060 281	0,000 000	0,716 79	0,283 21
665	0,112 210	0,044 096	0,000 000	0,717 89	0,282 11
670	0,081 261	0,031 800	0,000 000	0,718 73	0,281 27
675	0,057 930	0,022 602	0,000 000	0,719 34	0,280 66
680	0,040 851	0,015 905	0,000 000	0,719 76	0,280 24
685	0,028 623	0,011 130	0,000 000	0,720 02	0,279 98
690	0,019 941	0,007 749	0,000 000	0,720 16	0,279 84
695	0,013 842	0,005 375	0,000 000	0,720 30	0,279 70
700	0,009 577	0,003 718	0,000 000	0,720 36	0,279 64
705	0,006 605	0,002 565	0,000 000	0,720 32	0,279 68

λ,nm	$\bar{x}_{10}(\lambda)$	$\bar{y}_{10}(\lambda)$	$\bar{z}_{10}(\lambda)$	$x_{10}(\lambda)$	$y_{10}(\lambda)$
710	0,004 553	0,001 768	0,000 000	0,720 23	0,279 77
715	0,003 145	0,001 222	0,000 000	0,720 09	0,279 91
720	0,002 175	0,000 846	0,000 000	0,719 91	0,280 09
725	0,001 506	0,000 586	0,000 000	0,719 69	0,280 31
730	0,001 045	0,000 407	0,000 000	0,719 45	0,280 55
735	0,000 727	0,000 284	0,000 000	0,719 19	0,280 81
740	0,000 508	0,000 199	0,000 000	0,718 91	0,281 09
745	0,000 356	0,000 140	0,000 000	0,718 61	0,281 39
750	0,000 251	0,000 098	0,000 000	0,718 29	0,281 71
755	0,000 178	0,000 070	0,000 000	0,717 96	0,282 04
760	0,000 126	0,000 050	0,000 000	0,717 61	0,282 39
765	0,000 090	0,000 036	0,000 000	0,717 24	0,282 76
770	0,000 065	0,000 025	0,000 000	0,716 86	0,283 14
775	0,000 046	0,000 018	0,000 000	0,716 46	0,283 54
780	0,000 033	0,000 013	0,000 000	0,716 06	0,283 94

Summation at 5 nm intervals:

$$\sum \bar{x}_{10}(\lambda) = 23{,}329\,353 \qquad \sum \bar{y}_{10}(\lambda) = 23{,}332\,036 \qquad \sum \bar{z}_{10}(\lambda) = 23{,}334153 \text{ !!}$$

11.6 Table T.6. Relative spectral power distributions of illuminants representing typical fluorescent lamps, for wavelengths λ = 380 nm to 780 nm at 5 nm intervals

Table T.6.1. Fluorescent lamps FL1-12, already published in CIE 15.2 as F1-F12[XVIII].

FL1-6: standard; FL7-9: broad-band; FL10-12: narrow band fluorescent lamps.

λ, nm	FL1	FL2*	FL3	FL4	FL5	FL6	FL7*	FL8	FL9	FL10	FL11*	FL12
380	1,87	1,18	0,82	0,57	1,87	1,05	2,56	1,21	0,90	1,11	0,91	0,96
385	2,36	1,48	1,02	0,70	2,35	1,31	3,18	1,50	1,12	0,80	0,63	0,64
390	2,94	1,84	1,26	0,87	2,92	1,63	3,84	1,81	1,36	0,62	0,46	0,40
395	3,47	2,15	1,44	0,98	3,45	1,90	4,53	2,13	1,60	0,57	0,37	0,33
400	5,17	3,44	2,57	2,01	5,10	3,11	6,15	3,17	2,59	1,48	1,29	1,19
405	19,49	15,69	14,36	13,75	18,91	14,80	19,37	13,08	12,80	12,16	12,68	12,48
410	6,13	3,85	2,70	1,95	6,00	3,43	7,37	3,83	3,05	2,12	1,59	1,12
415	6,24	3,74	2,45	1,59	6,11	3,30	7,05	3,45	2,56	2,70	1,79	0,94
420	7,01	4,19	2,73	1,76	6,85	3,68	7,71	3,86	2,86	3,74	2,46	1,08
425	7,79	4,62	3,00	1,93	7,58	4,07	8,41	4,42	3,30	5,14	3,33	1,37
430	8,56	5,06	3,28	2,10	8,31	4,45	9,15	5,09	3,82	6,75	4,49	1,78
435	43,67	34,98	31,85	30,28	40,76	32,61	44,14	34,10	32,62	34,39	33,94	29,05
440	16,94	11,81	9,47	8,03	16,06	10,74	17,52	12,42	10,77	14,86	12,13	7,90

[XVIII] Illuminants FL2*, FL7* and FL11* should take priority over others when a few typical fluorescent lamp illuminants are to be selected.

λ, nm	FL1	FL2*	FL3	FL4	FL5	FL6	FL7*	FL8	FL9	FL10	FL11*	FL12
445	10,72	6,27	4,02	2,55	10,32	5,48	11,35	7,68	5,84	10,40	6,95	2,65
450	11,35	6,63	4,25	2,70	10,91	5,78	12,00	8,60	6,57	10,76	7,19	2,71
455	11,89	6,93	4,44	2,82	11,40	6,03	12,58	9,46	7,25	10,67	7,12	2,65
460	12,37	7,19	4,59	2,91	11,83	6,25	13,08	10,24	7,86	10,11	6,72	2,49
465	12,75	7,40	4,72	2,99	12,17	6,41	13,45	10,84	8,35	9,27	6,13	2,33
470	13,00	7,54	4,80	3,04	12,40	6,52	13,71	11,33	8,75	8,29	5,46	2,10
475	13,15	7,62	4,86	3,08	12,54	6,58	13,88	11,71	9,06	7,29	4,79	1,91
480	13,23	7,65	4,87	3,09	12,58	6,59	13,95	11,98	9,31	7,91	5,66	3,01
485	13,17	7,62	4,85	3,09	12,52	6,56	13,93	12,17	9,48	16,64	14,29	10,83
490	13,13	7,62	4,88	3,14	12,47	6,56	13,82	12,28	9,61	16,73	14,96	11,88
495	12,85	7,45	4,77	3,06	12,20	6,42	13,64	12,32	9,68	10,44	8,97	6,88
500	12,52	7,28	4,67	3,00	11,89	6,28	13,43	12,35	9,74	5,94	4,72	3,43
505	12,20	7,15	4,62	2,98	11,61	6,20	13,25	12,44	9,88	3,34	2,33	1,49
510	11,83	7,05	4,62	3,01	11,33	6,19	13,08	12,55	10,04	2,35	1,47	0,92
515	11,50	7,04	4,73	3,14	11,10	6,30	12,93	12,68	10,26	1,88	1,10	0,71
520	11,22	7,16	4,99	3,41	10,96	6,60	12,78	12,77	10,48	1,59	0,89	0,60
525	11,05	7,47	5,48	3,90	10,97	7,12	12,60	12,72	10,63	1,47	0,83	0,63
530	11,03	8,04	6,25	4,69	11,16	7,94	12,44	12,60	10,78	1,80	1,18	1,10
535	11,18	8,88	7,34	5,81	11,54	9,07	12,33	12,43	10,96	5,71	4,90	4,56
540	11,53	10,01	8,78	7,32	12,12	10,49	12,26	12,22	11,18	40,98	39,59	34,40
545	27,74	24,88	23,82	22,59	27,78	25,22	29,52	28,96	27,71	73,69	72,84	65,40
550	17,05	16,64	16,14	15,11	17,73	17,46	17,05	16,51	16,29	33,61	32,61	29,48
555	13,55	14,59	14,59	13,88	14,47	15,63	12,44	11,79	12,28	8,24	7,52	7,16
560	14,33	16,16	16,63	16,33	15,20	17,22	12,58	11,76	12,74	3,38	2,83	3,08
565	15,01	17,56	18,49	18,68	15,77	18,53	12,72	11,77	13,21	2,47	1,96	2,47
570	15,52	18,62	19,95	20,64	16,10	19,43	12,83	11,84	13,65	2,14	1,67	2,27
575	18,29	21,47	23,11	24,28	18,54	21,97	15,46	14,61	16,57	4,86	4,43	5,09
580	19,55	22,79	24,69	26,26	19,50	23,01	16,75	16,11	18,14	11,45	11,28	11,96
585	15,48	19,29	21,41	23,28	15,39	19,41	12,83	12,34	14,55	14,79	14,76	15,32
590	14,91	18,66	20,85	22,94	14,64	18,56	12,67	12,53	14,65	12,16	12,73	14,27
595	14,15	17,73	19,93	22,14	13,72	17,42	12,45	12,72	14,66	8,97	9,74	11,86
600	13,22	16,54	18,67	20,91	12,69	16,09	12,19	12,92	14,61	6,52	7,33	9,28
605	12,19	15,21	17,22	19,43	11,57	14,64	11,89	13,12	14,50	8,31	9,72	12,31
610	11,12	13,80	15,65	17,74	10,45	13,15	11,60	13,34	14,39	44,12	55,27	68,53
615	10,03	12,36	14,04	16,00	9,35	11,68	11,35	13,61	14,40	34,55	42,58	53,02
620	8,95	10,95	12,45	14,42	8,29	10,25	11,12	13,87	14,47	12,09	13,18	14,67
625	7,96	9,65	10,95	12,56	7,32	8,95	10,95	14,07	14,62	12,15	13,16	14,38
630	7,02	8,40	9,51	10,93	6,41	7,74	10,76	14,20	14,72	10,52	12,26	14,71
635	6,20	7,32	8,27	9,52	5,63	6,69	10,42	14,16	14,55	4,43	5,11	6,46
640	5,42	6,31	7,11	8,18	4,90	5,71	10,11	14,13	14,40	1,95	2,07	2,57
645	4,73	5,43	6,09	7,01	4,26	4,87	10,04	14,34	14,58	2,19	2,34	2,75
650	4,15	4,68	5,22	6,00	3,72	4,16	10,02	14,50	14,88	3,19	3,58	4,18

λ, nm	FL1	FL2*	FL3	FL4	FL5	FL6	FL7*	FL8	FL9	FL10	FL11*	FL12
655	3,64	4,02	4,45	5,11	3,25	3,55	10,11	14,46	15,51	2,77	3,01	3,44
660	3,20	3,45	3,80	4,36	2,83	3,02	9,87	14,00	15,47	2,29	2,48	2,81
665	2,81	2,96	3,23	3,69	2,49	2,57	8,65	12,58	13,20	2,00	2,14	2,42
670	2,47	2,55	2,75	3,13	2,19	2,20	7,27	10,99	10,57	1,52	1,54	1,64
675	2,18	2,19	2,33	2,64	1,93	1,87	6,44	9,98	9,18	1,35	1,33	1,36
680	1,93	1,89	1,99	2,24	1,71	1,60	5,83	9,22	8,25	1,47	1,46	1,49
685	1,72	1,64	1,70	1,91	1,52	1,37	5,41	8,62	7,57	1,79	1,94	2,14
690	1,67	1,53	1,55	1,70	1,48	1,29	5,04	8,07	7,03	1,74	2,00	2,34
695	1,43	1,27	1,27	1,39	1,26	1,05	4,57	7,39	6,35	1,02	1,20	1,42
700	1,29	1,10	1,09	1,18	1,13	0,91	4,12	6,71	5,72	1,14	1,35	1,61
705	1,19	0,99	0,96	1,03	1,05	0,81	3,77	6,16	5,25	3,32	4,10	5,04
710	1,08	0,88	0,83	0,88	0,96	0,71	3,46	5,63	4,80	4,49	5,58	6,98
715	0,96	0,76	0,71	0,74	0,85	0,61	3,08	5,03	4,29	2,05	2,51	3,19
720	0,88	0,68	0,62	0,64	0,78	0,54	2,73	4,46	3,80	0,49	0,57	0,71
725	0,81	0,61	0,54	0,54	0,72	0,48	2,47	4,02	3,43	0,24	0,27	0,30
730	0,77	0,56	0,49	0,49	0,68	0,44	2,25	3,66	3,12	0,21	0,23	0,26
735	0,75	0,54	0,46	0,46	0,67	0,43	2,06	3,36	2,86	0,21	0,21	0,23
740	0,73	0,51	0,43	0,42	0,65	0,40	1,90	3,09	2,64	0,24	0,24	0,28
745	0,68	0,47	0,39	0,37	0,61	0,37	1,75	2,85	2,43	0,24	0,24	0,28
750	0,69	0,47	0,39	0,37	0,62	0,38	1,62	2,65	2,26	0,21	0,20	0,21
755	0,64	0,43	0,35	0,33	0,59	0,35	1,54	2,51	2,14	0,17	0,24	0,17
760	0,68	0,46	0,38	0,35	0,62	0,39	1,45	2,37	2,02	0,21	0,32	0,21
765	0,69	0,47	0,39	0,36	0,64	0,41	1,32	2,15	1,83	0,22	0,26	0,19
770	0,61	0,40	0,33	0,31	0,55	0,33	1,17	1,89	1,61	0,17	0,16	0,15
775	0,52	0,33	0,28	0,26	0,47	0,26	0,99	1,61	1,38	0,12	0,12	0,10
780	0,43	0,27	0,21	0,19	0,40	0,21	0,81	1,32	1,12	0,09	0,09	0,05

Table T.6.2.a. New set of fluorescent lamps. FL3.1-3: Standard halophosphate lamps; FL3.4-6: DeLuxe type lamps; FL3.7-8: Three band fluorescent lamps.

λ, nm$^+$	FL3.1	FL3.2	FL3.3	FL3.4	FL3.5	FL3.6	FL3.7	FL3.8
380	2,39	5,80	8,94	3,46	4,72	5,53	3,79	4,18
385	2,93	6,99	11,21	3,86	5,82	6,63	2,56	2,93
390	3,82	8,70	14,08	4,41	7,18	8,07	1,91	2,29
395	4,23	9,89	16,48	4,51	8,39	9,45	1,42	1,98
400	4,97	11,59	19,63	4,86	9,96	11,28	1,51	2,44
405	86,30	94,53	116,33	71,22	58,86	61,47	73,64	70,70
410	11,65	20,80	32,07	8,72	15,78	17,80	7,37	10,19
415	7,09	16,52	29,72	5,36	15,10	17,47	4,69	9,79
420	7,84	18,30	33,39	5,61	17,30	20,12	5,33	13,21
425	8,59	20,33	36,94	5,91	19,66	23,05	6,75	17,79
430	9,44	22,00	40,33	6,42	22,43	26,37	8,51	22,98
435	196,54	231,90	262,66	192,77	176,00	186,01	181,81	191,43
440	10,94	25,81	46,87	7,77	28,67	33,94	11,71	31,76
445	11,38	27,63	49,79	8,37	31,92	37,98	11,96	33,35
450	11,89	29,10	52,46	9,22	35,38	42,12	12,18	33,87
455	12,37	30,61	54,81	10,18	38,73	46,38	11,90	32,89
460	12,81	31,92	56,81	11,18	41,98	50,30	11,16	30,60
465	13,15	33,11	58,44	12,28	44,92	53,95	11,22	28,28
470	13,39	33,83	59,52	13,38	47,49	56,94	9,83	24,81
475	13,56	34,70	60,12	14,54	49,58	59,48	8,94	21,60
480	13,59	35,02	60,24	15,74	51,21	61,36	12,08	23,40
485	13,56	35,22	59,88	17,09	52,36	62,68	52,56	68,99
490	14,07	35,81	59,88	19,60	53,99	64,34	55,42	70,85
495	13,39	35,14	58,60	21,05	53,78	63,90	31,69	42,29
500	13,29	35,14	57,85	23,96	54,04	63,85	16,03	22,67
505	13,25	34,90	56,29	27,77	53,88	63,24	6,72	11,08
510	13,53	34,70	54,81	32,68	53,62	62,46	4,59	7,66
515	14,24	35,02	53,42	38,29	53,25	61,41	3,67	6,07
520	15,74	36,13	52,70	43,76	53,09	60,47	3,02	5,07
525	18,26	37,92	52,50	47,72	52,88	59,48	3,21	4,88
530	22,28	40,62	53,30	50,27	52,99	58,65	4,90	6,26
535	27,97	44,70	54,89	51,78	53,15	57,93	19,05	20,29
540	35,70	49,63	57,61	52,68	53,67	57,49	177,64	204,67
545	148,98	154,16	182,75	167,36	167,93	175,17	347,34	390,25
550	56,55	62,21	65,27	55,29	55,61	57,27	116,80	135,69
555	68,68	68,92	69,41	56,94	56,82	57,49	31,87	34,57
560	79,99	75,83	73,28	59,30	58,39	57,99	16,37	15,71
565	91,47	81,95	76,56	62,15	60,22	58,76	14,92	12,60
570	101,32	86,95	78,67	65,26	62,21	59,64	14,12	11,05
575	123,16	103,54	95,74	84,26	81,45	78,77	29,50	25,05
580	129,53	109,94	97,22	89,22	84,96	81,26	61,40	54,98
585	115,05	91,95	76,79	75,79	68,71	63,18	85,05	82,84

λ, nm[+]	FL3.1	FL3.2	FL3.3	FL3.4	FL3.5	FL3.6	FL3.7	FL3.8
590	113,48	89,85	73,36	79,19	70,70	64,29	64,86	58,22
595	110,08	87,15	69,33	82,80	73,01	65,78	65,01	53,06
600	104,28	83,26	64,23	85,76	74,69	66,77	53,17	41,44
605	97,98	78,93	58,92	88,62	76,26	67,77	34,22	25,26
610	89,60	73,93	53,38	91,12	77,68	68,60	427,27	329,89
615	80,74	68,84	47,91	93,43	78,67	69,10	201,10	161,29
620	71,92	63,44	42,61	96,89	80,14	70,15	58,63	54,19
625	63,50	58,84	37,74	101,45	81,71	71,69	72,01	66,30
630	55,46	53,84	33,11	103,65	82,08	71,97	88,19	71,43
635	47,97	49,43	29,04	100,30	79,98	69,81	20,07	15,74
640	41,39	45,54	25,29	97,89	78,15	68,05	13,10	10,22
645	35,50	41,53	22,10	96,59	76,52	66,66	12,92	10,68
650	30,32	38,31	19,31	106,21	79,20	69,70	24,54	20,32
655	25,79	34,62	16,84	109,97	79,51	70,37	15,94	14,13
660	21,84	31,80	14,68	117,49	81,08	72,47	13,56	11,72
665	18,53	29,02	12,89	96,04	70,76	62,30	13,38	11,75
670	15,67	26,72	11,37	80,15	62,58	54,45	8,42	7,87
675	13,22	24,22	9,97	70,42	56,87	49,20	6,57	6,38
680	11,14	22,19	8,82	65,01	52,83	45,60	7,18	7,23
685	9,40	20,41	7,86	60,15	49,11	42,40	9,90	8,94
690	8,65	19,10	7,78	56,04	46,28	40,02	11,47	9,79
695	6,75	16,79	6,30	50,92	42,24	36,48	8,88	7,26
700	5,69	15,13	5,67	46,26	38,58	33,28	3,05	2,59
705	4,87	13,82	5,15	42,60	35,59	30,84	22,04	17,03
710	4,29	12,63	4,91	38,85	32,76	28,30	42,79	33,69
715	3,54	11,39	4,31	35,09	29,61	25,65	14,40	12,02
720	3,03	10,32	3,99	31,73	26,89	23,33	1,88	1,68
725	2,62	9,21	3,67	28,77	24,53	21,23	1,60	1,50
730	2,28	8,89	3,43	25,76	22,17	19,29	1,42	1,31
735	1,94	7,50	3,19	23,16	20,02	17,41	1,05	1,01
740	1,70	6,71	2,95	21,30	18,45	16,31	1,23	1,16
745	1,50	6,11	2,75	18,55	16,09	14,21	1,76	1,59
750	1,36	5,40	2,63	17,74	15,62	14,04	0,74	0,79
755	1,16	4,80	2,43	14,74	13,10	11,55	0,52	0,67
760	4,91	8,70	7,14	12,93	11,69	10,39	4,10	4,82
765	0,95	4,01	2,19	13,63	12,42	11,28	0,46	0,61
770	1,50	4,09	2,71	10,43	9,43	8,51	0,99	1,25
775	0,89	3,30	2,00	9,67	8,96	8,24	0,43	0,79
780	0,68	2,82	1,80	8,07	7,39	7,02	0,00	0,58

[+] This table gives the representative data of these lamps only in the 380 nm to 780 nm region for colorimetric purposes, though many fluorescent lamps emit power outside this spectral region, especially in the near ultraviolet. Other sources of data must be consulted if information is required below 380 nm or above 780 nm.

Table T.6.2b. New set of fluorescent lamps, cont. FL3.9-11: Three band fluorescent lamps; FL3.12-14: Multi-band fluorescent lamps; FL3.15: D65 simulator lamp.

λ, nm^{+}	FL3.9	FL3.10	FL3.11	FL3.12	FL3.13	FL3.14	FL3.15
380	3,77	0,25	3,85	1,62	2,23	2,87	300,00
385	2,64	0	2,91	2,06	2,92	3,69	286,00
390	2,06	0	2,56	2,71	3,91	4,87	268,00
395	1,87	0	2,59	3,11	4,55	5,82	244,00
400	2,55	0,69	3,63	3,67	5,46	7,17	304,00
405	71,68	21,24	74,54	74,60	77,40	72,21	581,00
410	12,05	2,18	14,69	8,88	11,25	13,69	225,00
415	13,57	1,86	17,22	4,77	7,69	11,12	155,00
420	19,60	3,1	24,99	4,72	8,29	12,43	152,00
425	27,33	5	34,40	4,72	8,98	13,90	170,00
430	35,39	7,03	44,57	4,94	10,01	15,82	295,00
435	211,82	45,08	228,08	150,29	204,45	200,99	1 417,00
440	49,02	16,78	61,53	6,08	13,75	21,72	607,00
445	51,83	12,28	65,31	7,13	16,88	26,33	343,00
450	52,50	13,31	66,35	9,10	21,73	32,85	386,00
455	50,73	13,66	64,37	11,76	27,96	40,80	430,00
460	46,93	13,69	59,81	14,96	34,92	49,23	469,00
465	42,42	13,13	54,24	18,54	41,96	57,39	502,00
470	37,16	12,28	47,42	22,48	48,62	65,26	531,00
475	31,84	11,42	41,10	26,76	54,33	71,99	552,00
480	31,94	11,66	40,04	31,66	59,49	78,25	567,00
485	77,74	22,04	85,54	40,93	67,91	88,85	572,00
490	79,45	26,17	86,55	45,83	70,01	91,67	575,00
495	47,93	18,57	53,47	46,00	66,40	86,81	561,00
500	26,24	11,36	30,91	45,26	62,07	80,42	548,00
505	13,15	6,83	17,41	43,16	56,95	73,82	527,00
510	8,80	5,58	12,56	41,63	52,70	69,12	507,00
515	6,70	4,88	10,10	39,75	48,54	63,69	482,00
520	5,38	4,31	8,48	37,83	44,80	58,44	461,00
525	4,93	3,76	7,74	36,16	41,75	53,57	438,00
530	6,06	3,61	8,58	35,25	39,77	49,66	418,00
535	19,76	5,62	21,39	37,04	40,50	48,44	404,00
540	215,94	38,59	220,12	59,86	59,27	72,56	429,00
545	412,13	100	417,35	183,53	184,09	200,42	1016,00
550	142,39	36,54	146,13	59,03	59,06	65,00	581,00
555	34,74	10,57	36,67	47,93	49,95	47,49	370,00
560	14,76	2,98	16,51	48,67	50,90	44,14	368,00
565	10,99	2,05	12,56	52,69	54,51	44,71	371,00
570	9,25	1,84	10,81	57,24	58,33	46,01	377,00
575	23,50	6,09	25,31	77,75	77,49	63,52	490,00
580	53,05	17,27	53,31	87,81	85,78	71,73	525,00
585	81,90	21,77	80,75	80,55	76,20	63,52	402,00

λ, nm[+]	FL3.9	FL3.10	FL3.11	FL3.12	FL3.13	FL3.14	FL3.15
590	54,92	18,72	53,56	84,83	78,73	64,13	404,00
595	47,80	10,15	44,02	86,84	78,95	63,74	412,00
600	36,65	7,26	33,05	91,44	81,48	66,82	418,00
605	21,82	5,17	20,26	96,51	84,57	70,65	425,00
610	285,69	56,66	233,61	105,25	87,75	79,29	428,00
615	139,94	49,39	118,20	106,74	89,56	80,77	432,00
620	53,37	18,57	51,66	108,53	91,36	83,59	433,00
625	64,30	14,21	61,27	106,92	89,00	82,59	431,00
630	64,04	14,01	55,15	101,54	83,67	77,60	427,00
635	13,79	5,99	12,95	95,20	78,26	72,47	420,00
640	9,06	2,68	8,93	89,34	73,19	68,34	410,00
645	9,83	3,14	9,77	82,95	67,61	63,82	399,00
650	18,60	6,25	17,12	75,78	61,42	58,57	385,00
655	13,38	5,78	13,01	68,65	55,49	53,18	370,00
660	10,99	6,75	10,45	61,70	49,78	47,97	352,00
665	10,77	5,16	10,33	55,23	44,46	43,14	336,00
670	7,57	3,03	7,70	48,58	39,13	38,19	317,00
675	6,19	1,57	6,34	42,90	34,45	33,85	298,00
680	7,09	1,72	7,35	37,74	30,28	29,94	277,00
685	8,54	1,54	8,22	32,93	26,37	26,24	260,00
690	8,77	1,71	7,93	29,65	23,88	23,90	242,00
695	6,41	1,1	5,70	25,19	20,10	20,33	223,00
700	2,26	0,28	2,23	21,69	17,40	17,42	202,00
705	15,02	3,65	12,43	19,28	15,29	15,64	187,00
710	29,39	7,54	24,24	17,36	13,62	14,34	167,00
715	10,22	2,34	8,74	14,74	11,68	12,21	152,00
720	1,42	0,05	1,39	12,86	10,31	10,65	136,00
725	1,23	0,04	1,23	11,28	9,11	9,43	125,00
730	1,10	0,04	1,10	9,97	8,03	8,34	113,00
735	0,84	0,03	0,84	8,88	7,13	7,52	103,00
740	0,97	0,03	0,94	7,78	6,31	6,73	93,00
745	1,35	0,02	1,23	7,04	5,67	6,08	84,00
750	0,65	0,02	0,68	6,30	5,11	5,52	75,00
755	0,13	0,01	0,52	5,55	4,55	5,00	66,00
760	4,22	0,01	4,60	10,15	9,06	9,47	58,00
765	0,10	0	0,45	4,50	3,74	4,08	51,00
770	0,68	0	1,04	4,81	4,04	4,43	46,00
775	0,16	0	0,45	3,72	3,14	3,39	41,00
780	0,00	0	0,00	3,28	2,75	3,17	37,00

[+] This table gives the representative data of these lamps only in the 380 nm to 780 nm region for colorimetric purposes, though many fluorescent lamps emit power outside this spectral region, especially in the near ultraviolet. Other sources of data must be consulted if information is required below 380 nm or above 780 nm.

11.7 Table T.7. High pressure discharge lamps. HP1: Standard high pressure sodium lamp; HP2: Colour enhanced high pressure sodium lamp; HP3-5: Three types of high pressure metal halide lamps

λ, nm$^+$	HP1	HP2		HP3	HP4	HP5
380	1,90	2,64		3,15	9,80	0,34
385	2,20	2,77		7,49	13,30	7,11
390	2,50	3,42		10,87	19,97	11,49
395	2,70	3,68		12,57	25,81	14,97
400	3,10	4,33		12,97	24,69	14,95
405	4,30	5,50		21,29	47,66	29,14
410	3,80	5,94		26,29	54,44	38,08
415	4,20	7,20		30,18	63,82	51,56
420	4,80	9,02		43,06	85,52	62,56
425	5,19	10,27		29,58	60,54	55,61
430	5,89	12,48		23,18	38,37	41,98
435	7,39	16,82		35,28	88,20	50,02
440	7,89	16,04		26,29	44,94	42,14
445	5,69	15,26		24,29	35,64	39,04
450	12,89	22,58		22,91	30,75	40,52
455	6,69	20,07		26,20	33,77	45,29
460	4,30	15,13		29,31	40,81	51,01
465	20,78	25,27		25,30	33,77	49,18
470	12,99	28,04		28,14	35,28	49,05
475	6,69	15,99		24,05	32,55	46,12
480	1,40	10,40		21,82	29,44	45,73
485	1,50	11,10		20,51	26,16	39,46
490	3,20	13,44		23,05	29,96	44,39
495	18,18	22,62		26,98	32,83	46,14
500	56,24	49,71		30,96	33,58	49,54
505	2,90	17,21		30,72	41,16	59,76
510	2,10	17,12		27,13	32,93	48,47
515	13,39	27,26		29,55	32,13	48,38
520	2,10	20,02		34,22	34,45	48,70
525	2,00	21,54		29,98	30,12	44,25
530	2,20	23,36		41,21	41,13	54,42
535	2,30	25,66		173,14	187,10	128,93
540	2,60	29,69		141,37	101,37	81,26
545	5,10	43,12		64,98	123,96	67,36
550	11,39	98,30		33,83	42,47	48,48
555	15,48	125,60		34,26	34,73	51,41
560	20,78	134,57		33,32	31,82	48,88
565	55,64	149,70		52,80	54,67	68,52
570	254,03	166,12		74,29	57,45	80,85
575	56,14	98,77		47,97	70,43	65,96
580	111,78	30,47		49,20	69,50	59,43

λ, nm[+]	HP1	HP2		HP3	HP4	HP5
585	297,98	1,17		96,07	49,37	67,57
590	142,55	0,39		85,41	183,35	128,34
595	334,84	1,65		175,18	162,15	131,85
600	189,40	21,41		153,73	109,35	101,70
605	117,78	76,11		120,22	72,38	77,05
610	79,92	126,16		98,90	70,60	66,27
615	108,09	161,96		90,22	58,08	77,09
620	46,85	160,06		70,07	44,13	60,51
625	38,16	158,19		66,84	50,20	65,23
630	32,47	153,69		57,61	40,80	57,86
635	28,37	147,40		53,03	37,91	56,20
640	25,37	140,60		49,85	36,71	54,32
645	22,98	134,92		48,16	38,30	56,34
650	20,38	127,59		42,76	31,24	45,74
655	19,78	124,65		50,64	35,31	50,79
660	17,78	118,02		48,42	45,62	56,66
665	16,78	113,94		41,27	35,82	51,99
670	19,18	118,10		43,44	89,91	84,31
675	17,98	115,16		40,48	36,01	47,48
680	13,69	102,85		35,16	32,57	47,46
685	9,99	90,54		34,94	39,26	61,78
690	8,19	83,34		24,68	23,27	34,51
695	7,59	79,44		24,70	25,30	38,74
700	6,99	76,97		21,49	20,02	30,98
705	6,79	74,85		19,49	17,54	25,45
710	6,49	73,12		18,48	16,25	22,88
715	6,39	71,51		17,55	15,20	20,82
720	6,09	70,13		17,36	15,15	21,05
725	5,99	69,04		17,09	15,22	20,81
730	5,79	67,48		16,32	14,26	18,69
735	5,79	66,70		16,07	12,63	17,54
740	5,79	66,31		16,58	14,75	19,58
745	5,79	65,14		15,78	13,19	16,42
750	6,39	65,70		17,66	17,63	23,77
755	5,99	64,79		20,46	23,38	35,39
760	5,59	64,10		16,59	16,02	21,37
765	31,97	83,04		17,81	24,46	34,58
770	27,87	86,25		16,07	22,05	30,21
775	5,89	63,93		14,83	16,11	19,71
780	6,69	64,92		14,61	12,91	15,61

[+] The relative spectral power distribution of the HP-illuminants given here can only be used over the spectral range from 380 nm to 780 nm. Although many high pressure gas discharge lamps emit power outside this spectral range, especially in the near ultraviolet, other sources of data must be consulted if information is required below 380 nm or above 780 nm.

11.8 Table T.8. Colorimetric data for the fluorescent lamp illuminants of Table T.6

Table T.8.1. Illuminants as published in Table T.6.1.

Lamp	Chromaticity coordinates		Correlated colour temperature, (T_{cp}) (kelvins)	General colour rendering index, R_a
	x	y		
FL 1	0,3131	0,3371	6430	76
FL 2	0,3721	0,3751	4230	64
FL 3	0,4091	0,3941	3450	57
FL 4	0,4402	0,4031	2940	51
FL 5	0,3138	0,3452	6350	72
FL 6	0,3779	0,3882	4150	59
FL 7	0,3129	0,3292	6500	90
FL 8	0,3458	0,3586	5000	95
FL 9	0,3741	0,3727	4150	90
FL 10	0,3458	0,3588	5000	81
FL 11	0,3805	0,3769	4000	83
FL12	0,4370	0,4042	3000	83

Table T.8.2. Illuminants as published in Table T.6.2.

	FL3.1	FL3.2	FL3.3	FL3.4	FL3.5	FL3.6	FL3.7
x=	0,4407	0,3808	0,3153	0,4429	0,3749	0,3488	0,4384
y=	0,4033	0,3734	0,3439	0,4043	0,3672	0,36	0,4045
T_{cp}=	2932 K	3965 K	6280 K	2904 K	4086 K	4894 K	2979 K
Special Rendering Indexes:							
No. 1 =	42	65	64	91	97	97	97
No. 2 =	69	80	80	89	97	97	94
No. 3 =	89	89	89	79	92	93	54
No. 4 =	39	66	69	88	94	97	88
No. 5 =	41	66	69	88	97	97	86
No. 6 =	52	71	74	82	95	95	81
No. 7 =	66	79	81	88	94	96	87
No. 8 =	13	48	49	89	94	96	64
No. 9 =	-109	-37	-63	76	88	93	-9
No. 10 =	29	51	52	69	90	90	51
No. 11 =	19	56	62	88	95	97	76
No. 12 =	21	59	68	63	90	92	50
No. 13 =	47	68	68	91	97	98	98
No. 14 =	93	94	93	87	95	95	69
General Colour Rendering Index							
R_a =	51	70	72	87	95	96	82

Table T.8.2 cont. Illuminants as published in Table T.6.2.

	FL3.8	FL3.9	FL3.10	FL3.11	FL3.12	FL3.13	FL3.14	FL3.15
x=	0,382	0,3499	0,3455	0,3245	0,4377	0,383	0,3447	0,3127
y=	0,3832	0,3591	0,356	0,3434	0,4037	0,3724	0,3609	0,3288
T_{cp}=	4006 K	4853 K	5000 K	5854 K	2984 K	3896 K	5045 K	6509 K
Special Rendering Indices								
No. 1 =	94	94	99	90	95	98	93	99
No. 2 =	89	89	97	86	98	97	94	99
No. 3 =	50	48	63	49	92	98	97	96
No. 4 =	85	84	92	82	95	97	94	98
No. 5 =	83	84	92	81	94	99	94	99
No. 6 =	73	72	85	70	97	97	93	100
No. 7 =	86	85	92	85	93	94	97	98
No. 8 =	72	78	86	79	83	88	97	98
No. 9 =	5	22	46	24	58	71	93	96
No. 10 =	40	38	62	34	88	99	91	99
No. 11 =	68	68	78	64	93	94	95	100
No. 12 =	48	51	72	50	85	89	85	95
No. 13 =	95	95	97	90	97	99	92	98
No. 14 =	67	66	75	67	94	98	97	98
General Colour Rendering Index								
R_a =	79	79	88	78	93	96	95	98

11.9 Table T.9. Colorimetric data for the high pressure illuminants of Table T.7

	HP1	HP2		HP3	HP4	HP5
x=	0,533	0,4778		0,4302	0,3812	0,3776
y=	0,415	0,4158		0,4075	0,3797	0,3713
T_{cp}=	1959 K	2506 K		3144 K	4002 K	4039 K
Special Rendering Indices						
No. 1 =	-3	98		87	75	87
No. 2 =	61	89		92	85	94
No. 3 =	40	73		87	84	97
No. 4 =	-27	89		89	78	89
No. 5 =	-4	88		85	75	89
No. 6 =	52	71		90	79	94
No. 7 =	21	81		82	77	85
No. 8 =	-75	72		50	42	64
No. 9 =	-260	52		-29	-60	10
No. 10 =	43	66		71	56	85
No. 11 =	-52	66		89	77	90
No. 12 =	27	55		72	64	90
No. 13 =	7	90		90	79	90
No. 14 =	61	82		91	91	98
General Colour Rendering Index						
Ra =	8	83		83	74	87

11.10 Table T.10. Values of the first deviation function used in the calculation of the observer metamerism index

λ, nm	$\Delta \bar{x}(\lambda)$	$\Delta \bar{y}(\lambda)$	$\Delta \bar{z}(\lambda)$
380	-0,0001	0,0000	-0,0002
385	-0,0003	0,0000	-0,0010
390	-0,0009	-0,0001	-0,0036
395	-0,0026	-0,0004	-0,0110
400	-0,0069	-0,0009	-0,0294
405	-0,0134	-0,0015	-0,0558
410	-0,0197	-0,0019	-0,0820
415	-0,0248	-0,0022	-0,1030
420	-0,0276	-0,0021	-0,1140
425	-0,0263	-0,0017	-0,1079
430	-0,0216	-0,0009	-0,0872
435	-0,0122	0,0005	-0,0455
440	-0,0021	0,0015	-0,0027
445	0,0036	0,0008	0,0171
450	0,0092	-0,0003	0,0342
455	0,0186	-0,0005	0,0703
460	0,0263	-0,0011	0,0976
465	0,0256	-0,0036	0,0859
470	0,0225	-0,0060	0,0641
475	0,0214	-0,0065	0,0547
480	0,0205	-0,0060	0,0475
485	0,0197	-0,0045	0,0397
490	0,0187	-0,0031	0,0319
495	0,0167	-0,0037	0,0228
500	0,0146	-0,0047	0,0150
505	0,0133	-0,0059	0,0117
510	0,0118	-0,0060	0,0096
515	0,0094	-0,0025	0,0062
520	0,0061	0,0010	0,0029
525	0,0017	0,0005	0,0005
530	-0,0033	-0,0011	-0,0012
535	-0,0085	-0,0020	-0, 0020
540	-0,0139	-0,0028	-0,0022
545	-0,0194	-0,0039	-0,0024
550	-0,0247	-0,0044	-0,0024
555	-0,0286	-0,0027	-0,0021
560	-0,0334	-0,0022	-0,0017
565	-0,0426	-0,0073	-0,0015
570	-0,0517	-0,0127	-0,0014
575	-0,0566	-0,0129	-0,0013
580	-0,0600	-0,0126	-0,0013
585	-0,0637	-0,0162	-0,0011
590	-0,0656	-0,0196	-0,0009
595	-0,0638	-0,0199	-0,0008
600	-0,0595	-0,0187	-0,0006
605	-0,0530	-0,0170	-0,0005
610	-0,0448	-0,0145	-0,0004
615	-0,0346	-0,0112	0,0000
620	-0,0242	-0,0077	0,0002
625	-0,0155	-0,0048	0,0000
630	-0,0085	-0,0025	-0,0002
635	-0,0044	-0,0012	-0,0002
640	-0,0019	-0,0006	0,0000
645	-0,0001	0,0000	0,0000
650	0,0010	0,0003	0,0000
655	0,0016	0,0005	0,0000
660	0,0019	0,0006	0,0000
665	0,0019	0,0006	0,0000
670	0,0017	0,0006	0,0000
675	0,0013	0,0005	0,0000
680	0,0009	0,0003	0,0000
685	0,0006	0,0002	0,0000
690	0,0004	0,0001	0,0000
695	0,0003	0,0001	0,0000
700	0,0002	0,0001	0,0000
705	0,0001	0,0000	0,0000
710	0,0001	0,0000	0,0000
715	0,0001	0,0000	0,0000
720	0,0000	0,0000	0,0000
725	0,0000	0,0000	0,0000
730	0,0000	0,0000	0,0000
735	0,0000	0,0000	0,0000
740	0,0000	0,0000	0,0000
745	0,0000	0,0000	0,0000
750	0,0000	0,0000	0,0000
755	0,0000	0,0000	0,0000
760	0,0000	0,0000	0,0000
765	0,0000	0,0000	0,0000
770	0,0000	0,0000	0,0000
775	0,0000	0,0000	0,0000
780	0,0000	0,0000	0,0000

APPENDIX A. OLD RECOMMENDATIONS, NOW OBSOLETE, AS WELL AS REFERENCES TO NON-CIE COLOUR DIFFERENCE FORMULAE

Appendix A.1. Illuminant B and Source B

Illuminant B

This illuminant was intended to represent direct sunlight with a correlated colour temperature of approximately 4900 K.

Source B

Illuminant B was realized by source A, combined with a filter consisting of a layer, one centimetre thick, of each of two solutions B_1 and B_2, contained in a double cell made of colourless optical glass. The solutions were made up as follows:

Solution B1:

Copper Sulphate ($CuS0_4 \cdot 5H_20$)	2,452	g
Mannite [$C_6H_8\,(OH)_6$]	2,452	g
Pyridine (C_5H_5N)	30,0	ml
Distilled water to make	1000,0	ml

Solution B2:

Cobalt Ammonium Sulphate [$CoSO_4$) $(NH_4)_2\,S0_4 \cdot 6H_2\,0$)]	21,71	g
Copper Sulphate ($CuSO_4\,. \cdot 5H_2\,0$)	16,11	g
Sulphuric Acid (density 1,835 $g{\cdot}ml^{-1}$)	10,0	ml
Distilled water to make	1000,0	ml

Appendix A.2. Illuminant C and Source C

Illuminant C

Representing average daylight with a correlated colour temperature of about 6800 K. The spectral power distribution of Illuminant C is reproduced in Table T.1.

Note: In Table T.1 the values for 775 nm and 780 nm have been added by extrapolation.

Source C

Illuminant C is to be realized by source A, combined with a filter consisting of a layer, one centimetre thick, of each of two solutions C_1 and C_2, contained in a double cell made of colourless optical glass. The solutions are to be made up as follows:

Solution C1:

Copper Sulphate ($CuSO_4 \cdot 5H_20$)	3,412	g
Mannite $C_6H_8\,(OH)_6$	3,412	g
Pyridine (C_5H_5N)	30,0	ml
Distilled water to make	1000,0	ml

Solution C2:

Cobalt Ammonium Sulphate [$CoS0_4\,.\,(NH_4)_2S0_4\,.\,6H_20$]	30,58	g
Copper Sulphate ($CuS0_4\,.\,5H_20$)	22,52	g
Sulphuric Acid (density 1,835 $g{\cdot}ml^{-1}$)	10,0	ml
Distilled water to make	1000,0	ml

Appendix A.3. CIE 1964 uniform colour space and colour difference formula

u, v uniform chromaticity scale (CIE 1960 UCS) diagram

This approximately uniform chromaticity diagram was produced by plotting $u = 4X/(X + 15Y + 3Z)$ as abscissa and $v = 6Y/(X + 15Y + 3Z)$ as ordinate; u and v are related to u' and v' of the CIE 1976 UCS diagram by the equations $u = u'$ and $v = 2v'/3$.

1964 uniform space and colour difference formula

This approximately uniform colour space was produced by plotting along three orthogonal axes

$$W^* = 25Y^{1/3} - 17$$

$$U^* = 13W^* (u - u_n)$$

$$V^* = 13W^* (v - v_n)$$

The associated colour difference formula was:

$$\Delta E = [(U^*_1 - U^*_2)^2 + (V^*_1 - V^*_2)^2 + (W^*_1 - W^*_2)^2]^{1/2}$$

The coordinates in this system bear the following approximate relationships to the coordinates of the CIELUV space:

$$W^* \cong L^*$$

$$U^* \cong u^*$$

$$V^* \cong 2/3\ v^*$$

Appendix A.4. CIE 1994 colour difference formula (CIE94)

Based on the work documented in CIE 101-1993 (CIE, 1993), where experiments are described that pointed out that a number of external parameters of a visual task affect the correlation of visual magnitude judgements of colour differences with their colorimetric measures, CIE started investigations with carefully selected data sets and reported a new colour-difference formula that introduced weighting factors to the lightness, chroma and hue differences, ΔL^*, ΔC^*_{ab} and ΔH^*_{ab}, of the CIELAB - formula. The resulting recommendation was as follows (CIE, 1995):

$$\Delta E^*_{94} = \left[\left(\frac{\Delta L^*}{k_L S_L}\right)^2 + \left(\frac{\Delta C^*_{ab}}{k_C S_C}\right)^2 + \left(\frac{\Delta H^*_{ab}}{k_H S_H}\right)^2\right]^{1/2}$$

The weighting functions, S_L, S_C, S_H adjust the internal non-uniform structure of the CIELAB - formula using

$$S_L = 1$$

$$S_C = 1 + 0{,}045\ C^*_{ab}$$

$$S_H = 1 + 0{,}015\ C^*_{ab}$$

If the standard of a sample pair is not clearly defined, C^*_{ab} may be replaced by the geometric mean $(C^*_{ab,1} \cdot C^*_{ab,2})^{1/2}$.

Note 1: The parametric factors, k_L, k_C, k_H are correction terms for variation in experimental conditions. Under reference conditions they are all set at 1. For other choices see (CIE, 1993).

Note 2: An alternative colour difference formula used by some ISO committees, but never endorsed by CIE is briefly described in Appendix A.5.

Note 3: The CIE 1994 colour difference formula is now obsolete and has been superseded by the CIEDE2000 formula.

Appendix A.5. CMC(l:c) colour difference formula

The Colour Measurement Committee of the Society of Dyers and Colourists (UK) recommended a colour difference formula that has been integrated into some ISO standards. It is a forerunner to CIE94 and was a model for developing the CIE94 formula. The main deviations from CIE94 are found in the weighting factors that are much more complicated mathematically in that they contain hue-dependent correction terms. They are defined as follows (Clarke et al., 1984):

$$S_L = 0{,}040\,975\, L_1^*/(1 + 0{,}017\,65\, L_1^*),\ \text{unless } L_1^* < 16 \text{ when } S_L = 0{,}511$$

$$S_C = 0{,}063\,8\, C^*_{ab,1}/(1 + 0{,}013\,1\, C^*_{ab,1}) + 0{,}638$$

$$S_H = S_C\,(Tf + 1 - f)$$

where

$$f = \{(C^*_{ab,1})^4/[(C^*_{ab,1})^4 + 1900]\}^{1/2}$$

$$T = 0{,}36 + |0{,}4 \cos(h_{ab,1} + 35)|$$

unless $h_{ab,1}$ is between 164° and 345° when

$$T = 0{,}56 + |0{,}2 \cos(h_{ab,1} + 168)|$$

The parametric factors are defined as follows:

$$k_L = l$$

$$k_C = c$$

$$k_H = 1$$

The parametric factors are mostly chosen $c = 1$ and l varied between 1 and 2. The choice of l and c must be indicated by setting the right numbers in the name of the formula, e.g. for textiles a choice of CMC(2:1) is in common use.

CIE recommends the use of the CIEDE2000 formula whenever in the past the CIE 94 or CMC formula were used[XIX].

Appendix A.6. DIN99 colour difference formula

The preceding colour difference formulae including the CIEDE2000 colour difference formula are non-vectorial transformations from CIELAB space, and do not define a space for colour differences. A new formula for colour differences was developed to cope with this problem: the DIN99 colour difference formula published in DIN 6176 *Colorimetric determination of colour differences of surface colours using the DIN 99 formula* (DIN, 2003).

References

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[XIX] The recommended use of CIEDE2000 is in agreement with the persons who developed the CMC formula.

APPENDIX B. DEFINITIONS OF THE $\bar{r}(\lambda), \bar{g}(\lambda), \bar{b}(\lambda)$ COLOUR-MATCHING FUNCTIONS, THE CIE RGB SYSTEM AND THE DERIVATION OF THE CIE XYZ SYSTEM FROM THE CIE RGB SYSTEM FOR THE 1931 STANDARD OBSERVER

The colour-matching functions $\bar{x}(\lambda), \bar{y}(\lambda), \bar{z}(\lambda)$ were originally derived from colour-matching functions $\bar{r}(\lambda), \bar{g}(\lambda), \bar{b}(\lambda)$ referring to spectral reference stimuli [R], [G], [B]. These reference stimuli were specified as stimuli of wavelengths 700,0 nm, 546,1 nm, and 435,8 nm, respectively. Their units were chosen to make a mixture of equal quantities of the three reference stimuli match the equi-energy spectrum. The luminances of the units of the three spectral stimuli were in the ratios 1,0000 : 4,5907 : 0,0601.

Appendix B.1. Determination of the $\bar{r}(\lambda), \bar{g}(\lambda), \bar{b}(\lambda)$ colour-matching functions

This determination results from two conventions originally adopted in 1931.

The first convention was to adopt the ratios of the luminance of the reference colour stimuli [R], [G], [B] as values.

The second convention was to attribute to the monochromatic stimuli constituting the colour-matching functions a luminance equal to the spectral luminous efficiency $V(\lambda)$.

It follows from these conventions that

$$1{,}0000\ \bar{r}(\lambda) + 4{,}5907\ \bar{g}(\lambda) + 0{,}0601\ \bar{b}(\lambda) = V(\lambda) \qquad \text{B.1}$$

Since for any stimulus

$$R/r = G/g = B/b = L/l \qquad \text{B.2}$$

where l is the characteristic luminance of the chromaticity whose expression for monochromatic stimuli reads

$$l(\lambda) = 1{,}00000\ r(\lambda) + 4{,}5907\ g(\lambda) + 0{,}0601\ b(\lambda) \qquad \text{B.3}$$

it follows that, with $L = V(\lambda)$

$$\bar{r}(\lambda) = [r(\lambda)\cdot V(\lambda)]/l(\lambda),\ \bar{g}(\lambda) = [g(\lambda)\cdot V(\lambda)]/l(\lambda),\ \bar{b}(\lambda) = [b(\lambda)\cdot V(\lambda)]/l(\lambda), \qquad \text{B.4}$$

These colour-matching functions as well as the corresponding chromaticity coordinates, are given in Table B.1.

The colour-matching functions $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, $\bar{z}(\lambda)$ given in Table T.4 agree closely with those defined originally in 1931. Three minor changes have been introduced. If rounded to four decimal places, at λ =775 nm the new value of $\bar{x}(\lambda)$ is 0,0001 instead of 0,0000; at λ = 555 nm $\bar{y}(\lambda)$ is 1,0000 instead of 1,0002; and at λ = 740 nm $\bar{y}(\lambda)$ is 0,0002 instead of 0,0003. These changes are considered insignificant in most colorimetric calculations.

It has been realized at an early stage that the 1924 $V(\lambda)$ function is too low in the blue part of the spectrum. A corrected $V_M(\lambda)$ function has been officially accepted by the CIE in 1988 (CIE, 1990). However, the $X(\lambda)$, $Y(\lambda)$, $Z(\lambda)$ colour-matching functions have not been modified. A CIE Technical Committee is currently investigating the question of obtaining visually meaningful colour-matching functions, see e.g. (Schanda, 1998).

Appendix B.2. Derivation of the CIE XYZ trichromatic system from the CIE RGB trichromatic system

The derivation of the XYZ system from the RGB system had to fulfil a certain number of requirements. Their fulfilment led to the establishment of a set of three linear equations relating the reference stimuli [X], [Y], [Z] to the reference stimuli [R], [G], [B] as follows:

$$\begin{aligned} [\mathrm{X}] &= 0{,}418\,455\,[\mathrm{R}] && - 0{,}091\,165\,[\mathrm{G}] && + 0{,}000\,921\,[\mathrm{B}] \\ [\mathrm{Y}] &= -0{,}158\,657\,[\mathrm{R}] && + 0{,}252\,426\,[\mathrm{G}] && - 0{,}002\,550\,[\mathrm{B}] \\ [\mathrm{Z}] &= -0{,}082\,832\,[\mathrm{R}] && + 0{,}015\,705\,[\mathrm{G}] && + 0{,}178\,595\,[\mathrm{B}] \end{aligned} \tag{B.5}$$

Another set of three linear equations yield, for any stimulus, the tristimulus values *X, Y, Z* as function of the tristimulus values *R, G, B*:

$$\begin{aligned} X &= 2{,}768\,892\,R && + 1{,}751\,748\,G && + 1{,}130\,160\,B \\ Y &= 1{,}000\,000\,R && + 4{,}590\,700\,G && + 0{,}060\,100\,B \\ Z &= 0 && + 0{,}056\,508\,G && + 5{,}594\,292\,B \end{aligned} \tag{B.6}$$

These sets B. 5 and B. 6 are not independent, each one may be mathematically derived from the other. The set B. 6 was developed first and its coefficients are exact values (Morren, 2000). This set may be considered as more appropriate in the present context, but the set B. 5 is more basic. Since colour-matching functions are a particular case of tristimulus values, the set B. 6 gives directly three equations providing $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, $\bar{z}(\lambda)$ as a function of $\bar{r}(\lambda), \bar{g}(\lambda), \bar{b}(\lambda)$.

The second of these equations:

$$\bar{y}(\lambda) = 1{,}000\,000\;\bar{r}(\lambda) + 4{,}590\,700\;\bar{g}(\lambda) + 0{,}060\,100\;\bar{b}(\lambda)$$

is particularly interesting: it reproduces the left-hand side of equation B. 1 under Section B. 1 and thus immediately leads to $\bar{y}(\lambda) = V(\lambda)$.

Note: However, the same conclusion may be obtained by considering other features of the XYZ system. The transformation between reference stimuli attributes to the luminance L_Y the value 1. Moreover, since the reference stimuli [X] and [Z] are located on the alychne of the system, leading to $L_X = L_Z = 0$, one has for any stimulus

$$L = Y \text{ and } l = y \tag{B.7}$$

Consequently, equations similar to B. 4 lead to

$$\bar{x}(\lambda) = [x(\lambda)\cdot V(\lambda)]/y(\lambda),\quad \bar{y}(\lambda) = V(\lambda),\quad \bar{z}(\lambda) = [z(\lambda)\cdot V(\lambda)]/y(\lambda) \tag{B.8}$$

These expressions are simpler than those given by the set B. 6.

In deriving the CIE XYZ colorimetric system a fundamental criterion was to link the system to photometry. The units of the RGB system, the RGB primaries, were already determined by their luminance. Furthermore this was done by using a transformation from the RGB system to the XYZ system where $\bar{y}(\lambda)$ became equivalent to $V(\lambda)$, determined by flicker photometry. Although $\bar{y}(\lambda)$ was intended to be a brightness correlate, it is actually a luminance correlate, i.e. it does not describe the brightness relationship between coloured lights. It does, however, provide a good estimate for the visibility of fine detail.

Appendix B.3. Definition of the colour-matching functions in the CIE 1964 trichromatic system

The colour-matching functions $\bar{x}_{10}(\lambda)$, $\bar{y}_{10}(\lambda)$, $\bar{z}_{10}(\lambda)$ defining the CIE 1964 standard colorimetric observer given in Table T.5 were derived from colour-matching functions referring to matching stimuli $[R_{10}]$, $[G_{10}]$, $[B_{10}]$. These are stimuli specified in terms of wavenumbers (ν) 15 500 cm^{-1}, 19 000 cm^{-1}, and 22 500 cm^{-1} (corresponding approximately to wavelengths 645,2 nm, 526,3 nm, and 444,4 nm), and their amounts are given in power units.

The three reference stimuli $[R_{10}]$, $[G_{10}]$, $[B_{10}]$ have the same power and the colour-matching functions $\bar{r}_{10}(\nu)$, $\bar{g}_{10}(\nu)$, $\bar{b}_{10}(\nu)$ are defined as monochromatic stimuli whose power equates to that of the reference stimuli. The $\bar{r}_{10}(\nu)$, $\bar{g}_{10}(\nu)$, $\bar{b}_{10}(\nu)$ colour matching functions are reproduced in Table B.2.

The derivation of the $[X_{10}]$, $[Y_{10}]$, $[Z_{10}]$ system from the $[R_{10}]$, $[G_{10}]$, $[B_{10}]$ system is based on principles that lead to a coordinate system similar to the system associated with the CIE 1931 standard colorimetric observer. The following transformation equations relate very closely the $\bar{r}_{10}(\nu)$, $\bar{g}_{10}(\nu)$, $\bar{b}_{10}(\nu)$ values of Table B.2 to $\bar{x}_{10}(\nu)$, $\bar{y}_{10}(\nu)$, $\bar{z}_{10}(\nu)$ values:

$$\bar{x}_{10}(\nu) = 0{,}341\,080\,\bar{r}_{10}(\nu) + 0{,}189\,145\,\bar{g}_{10}(\nu) + 0{,}387\,529\,\bar{b}_{10}(\nu)$$

$$\bar{y}_{10}(\nu) = 0{,}139\,058\,\bar{r}_{10}(\nu) + 0{,}837\,460\,\bar{g}_{10}(\nu) + 0{,}073\,316\,\bar{b}_{10}(\nu)$$

$$\bar{z}_{10}(\nu) = 0{,}000\,000\,\bar{r}_{10}(\nu) + 0{,}039\,553\,\bar{g}_{10}(\nu) + 2{,}026\,200\,\bar{b}_{10}(\nu)$$

Chromaticity coordinates $x_{10}(\nu)$, $y_{10}(\nu)$, $z_{10}(\nu)$ were then computed from

$$x_{10}(\nu) = \frac{\bar{x}_{10}(\nu)}{\bar{x}_{10}(\nu) + \bar{y}_{10}(\nu) + \bar{z}_{10}(\nu)}$$

$$y_{10}(\nu) = \frac{\bar{y}_{10}(\nu)}{\bar{x}_{10}(\nu) + \bar{y}_{10}(\nu) + \bar{z}_{10}(\nu)}$$

$$z_{10}(\nu) = \frac{\bar{z}_{10}(\nu)}{\bar{x}_{10}(\nu) + \bar{y}_{10}(\nu) + \bar{z}_{10}(\nu)}$$

Table 2 of CIE standard colorimetric observers (CIE, 1986) contains the official colour-matching functions $\bar{x}_{10}(\lambda)$, $\bar{y}_{10}(\lambda)$, $\bar{z}_{10}(\lambda)$ and chromaticity coordinates $x_{10}(\lambda)$, $y_{10}(\lambda)$, $z_{10}(\lambda)$ on a wavelength basis obtained by interpolation and extrapolation of the functions $\bar{x}_{10}(\lambda)$, $\bar{y}_{10}(\lambda)$, $\bar{z}_{10}(\lambda)$ and $x_{10}(\lambda)$, $y_{10}(\lambda)$, $z_{10}(\lambda)$ at intervals of 1 nm. Table T.5 contains every fifth value of this table.

The above transformation equations deviate somewhat from those originally published (CIE, 1959), which related the unsmoothed values of $\bar{r}_{10}(\nu)$, $\bar{g}_{10}(\nu)$, $\bar{b}_{10}(\nu)$ with the unsmoothed values of $\bar{x}_{10}(\lambda)$, $\bar{y}_{10}(\lambda)$, $\bar{z}_{10}(\lambda)$. The values published in CIE 15.2 (Table 2.6) are reproduced in this Appendix as Table B. 2.

In deriving the CIE 1964 standard colorimetric observer no direct photometric measurements were required. The large-field colour matching data as defined by the CIE 1964 standard colorimetric observer are intended to apply to matches where the luminance and relative spectral power distributions of the matched stimuli are such that no participation of the rod receptors of the visual mechanism is to be expected. This condition of observation is important as "rod intrusion" may upset the predictions of the standard observer.

In obtaining the RGB-XYZ transformation CIE tried to get the $\bar{y}_{10}(\lambda)$ function as near as possible to a 10 degree spectral luminosity function based on flicker photometry. The CIE is currently investigating the possibility of introducing on this basis a $V_{10}(\lambda)$ function.

Table B.1. Colour-matching functions $\bar{r}(\lambda), \bar{g}(\lambda), \bar{b}(\lambda)$ and corresponding chromaticity coordinates $r(\lambda)$, $g(\lambda)$, $b(\lambda)$ for wavelength 380 nm to 780 nm at 5 nm intervals for the 1931 standard observer.

λ, nm	$\bar{r}(\lambda)$	$\bar{g}(\lambda)$	$\bar{b}(\lambda)$	$r(\lambda)$	$g(\lambda)$	$b(\lambda)$
380	0,00003	-0,00001	0,00117	0,0272	-0,0115	0,9843
385	0,00005	-0,00002	0,00189	0,0268	-0,0114	0,9846
390	0,00010	-0,00004	0,00359	0,0263	-0,0114	0,9851
395	0,00017	-0,00007	0,00647	0,0256	-0,0113	0,9857
400	0,00030	-0,00014	0,01214	0,0247	-0,0112	0,9865
405	0,00047	-0,00022	0,01969	0,0237	-0,0111	0,9874
410	0,00084	-0,00041	0,03707	0,0225	-0,0109	0,9884
415	0,00139	-0,00070	0,06637	0,0207	-0,0104	0,9897
420	0,00211	-0,00110	0,11541	0,0181	-0,0094	0,9913
425	0,00266	-0,00143	0,18575	0,0142	-0,0076	0,9934
430	0,00218	-0,00119	0,24769	0,0088	-0,0048	0,9960
435	0,00036	-0,00021	0,29012	0,0012	-0,0007	0,9995
440	-0,00261	0,00149	0,31228	-0,0084	0,0048	1,0036
445	-0,00673	0,00379	0,31860	-0,0213	0,0120	1,0093
450	-0,01213	0,00678	0,31670	-0,0390	0,0218	1,0172
455	-0,01874	0,01046	0,31166	-0,0618	0,0345	1,0273
460	-0,02608	0,01485	0,29821	-0,0909	0,0517	1,0392
465	-0,03324	0,01977	0,27295	-0,1281	0,0762	1,0519
470	-0,03933	0,02538	0,22991	-0,1821	0,1175	1,0646
475	-0,04471	0,03183	0,18592	-0,2584	0,1840	1,0744
480	-0,04939	0,03914	0,14494	-0,3667	0,2906	1,0761
485	-0,05364	0,04713	0,10968	-0,5200	0,4568	1,0632
490	-0,05814	0,05689	0,08257	-0,7150	0,6996	1,0154
495	-0,06414	0,06948	0,06246	-0,9459	1,0247	0,9212
500	-0,07173	0,08536	0,04776	-1,1685	1,3905	0,7780
505	-0,08120	0,10593	0,03688	-1,3182	1,7195	0,5987
510	-0,08901	0,12860	0,02698	-1,3371	1,9318	0,4053
515	-0,09356	0,15262	0,01842	-1,2076	1,9699	0,2377
520	-0,09264	0,17468	0,01221	-0,9830	1,8534	0,1296
525	-0,08473	0,19113	0,00830	-0,7386	1,6662	0,0724
530	-0,07101	0,20317	0,00549	-0,5159	1,4761	0,0398
535	-0,05316	0,21083	0,00320	-0,3304	1,3105	0,0199
540	-0,03152	0,21466	0,00146	-0,1707	1,1628	0,0079
545	-0,00613	0,21487	0,00023	-0,0293	1,0282	0,0011
550	0,02279	0,21178	-0,00058	0,0974	0,9051	-0,0025
555	0,05514	0,20588	-0,00105	0,2121	0,7919	-0,0040
560	0,09060	0,19702	-0,00130	0,3164	0,6881	-0,0045
565	0,12840	0,18522	-0,00138	0,4112	0,5932	-0,0044
570	0,16768	0,17087	-0,00135	0,4973	0,5067	-0,0040
575	0,20715	0,15429	-0,00123	0,5751	0,4283	-0,0034
580	0,24526	0,13610	-0,00108	0,6449	0,3579	-0,0028

λ, nm	$\bar{r}(\lambda)$	$\bar{g}(\lambda)$	$\bar{b}(\lambda)$	$r(\lambda)$	$g(\lambda)$	$b(\lambda)$
585	0,27989	0,11686	-0,00093	0,7071	0,2952	-0,0023
590	0,30928	0,09754	-0,00079	0,7617	0,2402	-0,0019
595	0,33184	0,07909	-0,00063	0,8087	0,1928	-0,0015
600	0,34429	0,06246	-0,00049	0,8475	0,1537	-0,0012
605	0,34756	0,04776	-0,00038	0,8800	0,1209	-0,0009
610	0,33971	0,03557	-0,00030	0,9059	0,0949	-0,0008
615	0,32265	0,02583	-0,00022	0,9265	0,0741	-0,0006
620	0,29708	0,01828	-0,00015	0,9425	0,0580	-0,0005
625	0,26348	0,01253	-0,00011	0,9550	0,0454	-0,0004
630	0,22677	0,00833	-0,00008	0,9649	0,0354	-0,0003
635	0,19233	0,00537	-0,00005	0,9730	0,0272	-0,0002
640	0,15968	0,00334	-0,00003	0,9797	0,0205	-0,0002
645	0,12905	0,00199	-0,00002	0,9850	0,0152	-0,0002
650	0,10167	0,00116	-0,00001	0,9888	0,0113	-0,0001
655	0,07857	0,00066	-0,00001	0,9918	0,0083	-0,0001
660	0,05932	0,00037	0,00000	0,9940	0,0061	-0,0001
665	0,04366	0,00021	0,00000	0,9954	0,0047	-0,0001
670	0,03149	0,00011	0,00000	0,9966	0,0035	-0,0001
675	0,02294	0,00006	0,00000	0,9975	0,0025	0,0000
680	0,01687	0,00003	0,00000	0,9984	0,0016	0,0000
685	0,01187	0,00001	0,00000	0,9991	0,0009	0,0000
690	0,00819	0,00000	0,00000	0,9996	0,0004	0,0000
695	0,00572	0,00000	0,00000	0,9999	0,0001	0,0000
700	0,00410	0,00000	0,00000	1,0000	0,0000	0,0000
705	0,00291	0,00000	0,00000	1,0000	0,0000	0,0000
710	0,00210	0,00000	0,00000	1,0000	0,0000	0,0000
715	0,00148	0,00000	0,00000	1,0000	0,0000	0,0000
720	0,00105	0,00000	0,00000	1,0000	0,0000	0,0000
725	0,00074	0,00000	0,00000	1,0000	0,0000	0,0000
730	0,00052	0,00000	0,00000	1,0000	0,0000	0,0000
735	0,00036	0,00000	0,00000	1,0000	0,0000	0,0000
740	0,00025	0,00000	0,00000	1,0000	0,0000	0,0000
745	0,00017	0,00000	0,00000	1,0000	0,0000	0,0000
750	0,00012	0,00000	0,00000	1,0000	0,0000	0,0000
755	0,00008	0,00000	0,00000	1,0000	0,0000	0,0000
760	0,00006	0,00000	0,00000	1,0000	0,0000	0,0000
765	0,00004	0,00000	0,00000	1,0000	0,0000	0,0000
770	0,00003	0,00000	0,00000	1,0000	0,0000	0,0000
775	0,00001	0,00000	0,00000	1,0000	0,0000	0,0000
780	0,00000	0,00000	0,00000	1,0000	0,0000	0,0000

Table B.2. CIE 1964 standard colorimetric observer. Colour-matching functions $\bar{r}_{10}(\nu)$, $\bar{g}_{10}(\nu)$, $\bar{b}_{10}(\nu)$ and corresponding chromaticity coordinates $r_{10}(\nu)$, $g_{10}(\nu)$, $b_{10}(\nu)$ for wavenumbers ν = 27 750 to 12 250 cm^{-1} at 250 cm^{-1} intervals.

ν, cm^{-1}	$\bar{r}_{10}(\nu)$	$\bar{g}_{10}(\nu)$	$\bar{b}_{10}(\nu)$	$r_{10}(\nu)$	$g_{10}(\nu)$	$b_{10}(\nu)$
27750	0,000000079100	-0,000000021447	0,000000307299	0,21674	-0,05877	0,84203
27500	0,00000029891	-0,00000008125	0,00000116475	0,21622	-0,05877	0,84255
27250	0,00000108348	-0,00000029533	0,00000423733	0,21560	-0,05877	0,84317
27000	0,0000037522	-0,0000010271	0,0000147506	0,21471	-0,05877	0,84406
26750	0,0000123776	-0,0000034057	0,0000489822	0,21358	-0,05877	0,84519
26500	0,000038728	-0,000010728	0,000154553	0,21215	-0,05877	0,84662
26250	0,000114541	-0,000032004	0,000462055	0,21032	-0,05877	0,84844
26000	0,00031905	-0,00009006	0,00130350	0,20819	-0,05877	0,85058
25750	0,00083216	-0,00023807	0,00345702	0,20542	-0,05877	0,85335
25500	0,00201685	-0,00058813	0,00857776	0,20155	-0,05877	0,85722
25250	0,0045233	-0,0013519	0,0198315	0,19664	-0,05877	0,86213
25000	0,0093283	-0,0028770	0,0425057	0,19054	-0,05877	0,86823
24750	0,0176116	-0,0056200	0,0840402	0,18339	-0,05852	0,87513
24500	0,030120	-0,010015	0,152451	0,17455	-0,05804	0,88349
24250	0,045571	-0,016044	0,251453	0,16219	-0,05710	0,89491
24000	0,060154	-0,022951	0,374271	0,14619	-0,05578	0,90959
23750	0,071261	-0,029362	0,514950	0,12797	-0,05273	0,92476
23500	0,074212	-0,032793	0,648306	0,10760	-0,04755	0,93995
23250	0,068535	-0,032357	0,770262	0,08498	-0,04012	0,95514
23000	0,055848	-0,027996	0,883628	0,06127	-0,03071	0,96944
22750	0,033049	-0,017332	0,965742	0,03367	-0,01766	0,98399
22500	0,000000	0,000000	1,000000	0,00000	0,00000	1,00000
22250	-0,041570	0,024936	0,987224	-0,04283	0,02569	1,01714
22000	-0,088073	0,057100	0,942474	-0,09662	0,06264	1,03398
21750	-0,143959	0,099886	0,863537	-0,17567	0,12189	1,05378
21500	-0,207995	0,150955	0,762081	-0,29501	0,21411	1,08090
21250	-0,285499	0,218942	0,630116	-0,50660	0,38850	1,11810
21000	-0,346240	0,287846	0,469818	-0,84156	0,69963	1,14193
20750	-0,388289	0,357723	0,333077	-1,28355	1,18251	1,10104
20500	-0,426587	0,435138	0,227060	-1,81056	1,84685	0,96371
20250	-0,435789	0,513218	0,151027	-1,90754	2,24646	0,66108
20000	-0,438549	0,614637	0,095840	-1,61274	2,26029	0,35245
19750	-0,404927	0,720251	0,057654	-1,08566	1,93108	0,15458
19500	-0,333995	0,830003	0,029877	-0,63511	1,57830	0,05681
19250	-0,201889	0,933227	0,012874	-0,27128	1,25398	0,01730
19000	0,000000	1,000000	0,000000	0,00000	1,00000	0,00000
18750	0,255754	1,042957	-0,008854	0,19828	0,80858	-0,00686
18500	0,556022	1,061343	-0,014341	0,34686	0,66209	-0,00895
18250	0,904637	1,031339	-0,017422	0,47152	0,53756	-0,00908
18000	1,314803	0,976838	-0,018644	0,57844	0,42976	-0,00820
17750	1,770322	0,887915	-0,017338	0,67035	0,33622	-0,00657
17500	2,236809	0,758780	-0,014812	0,75041	0,25456	-0,00497
17250	2,641981	0,603012	-0,011771	0,81714	0,18650	-0,00364
17000	3,002291	0,452300	-0,008829	0,87130	0,13126	-0,00256

ν, cm^{-1}	$\bar{r}_{10}(\nu)$	$\bar{g}_{10}(\nu)$	$\bar{b}_{10}(\nu)$	$r_{10}(\nu)$	$g_{10}(\nu)$	$b_{10}(\nu)$
16750	3,159249	0,306869	-0,005990	0,91304	0,08869	-0,00173
16500	3,064234	0,184057	-0,003593	0,94438	0,05673	-0,00111
16250	2,717232	0,094471	-0,001844	0,96704	0,03362	-0,00066
16000	2,191156	0,041693	-0,000815	0,98169	0,01868	-0,00037
15750	1,566864	0,013407	-0,000262	0,99168	0,00849	-0,00017
15500	1,000000	0,000000	0,000000	1,00000	0,00000	0,00000
15250	0,575756	-0,002747	0,000054	1,00470	-0,00479	0,00009
15000	0,296964	-0,002029	0,000040	1,00674	-0,00688	0,00014
14750	0,138738	-0,001116	0,000022	1,00795	-0,00811	0,00016
14500	0,0602209	-0,0005130	0,0000100	1,00842	-0,00859	0,00017
14250	0,0247724	-0,0002152	0,0000042	1,00859	-0,00876	0,00017
14000	0,00976319	-0,00008277	0,00000162	1,00838	-0,00855	0,00017
13750	0,00375328	0,00003012	0,00000059	1,00793	-0,00809	0,00016
13500	0,00141908	-0,00001051	0,00000021	1,00731	-0,00746	0,00015
13250	0,000533169	-0,000003543	0,000000069	1,00656	-0,00669	0,00013
13000	0,000199730	-0,000001144	0,000000022	1,00565	-0,00576	0,00011
12750	0,0000743522	-0,0000003472	0,0000000068	1,00460	-0,00469	0,00009
12500	0,0000276506	-0,0000000961	0,0000000019	1,00342	-0,00349	0,00007
12250	0,0000102123	-0,0000000220	0,0000000004	1,00212	-0,00216	0,00004

Reference

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APPENDIX C. ALTERNATIVE METHOD TO DEFINE DAYLIGHT ILLUMINANTS - METHOD OF CALCULATION, CORRECTED TABLES AND EQUATIONS (FOR INFORMATION AND EVALUATION)

If one tries to apply the calculation procedure for other illuminants D, which is described in Section 3.1 to D65, one has to follow a rounding process for computing the coefficients M_1 and M_2 and then a peculiar practice for rounding the final results. These unusual procedures explain the slight deviations sometimes observed in comparison with the official table for standard illuminant D65. These changes are negligible for colorimetric calculations, but are annoying if tables are compared, see (Kránicz and Schanda, 2000).

Daylight illuminant spectral power distribution tables are based on the work of Judd and co-workers (Judd et al., 1964), who proposed the calculation method of daylight illuminants based on the S_0, S_1, S_2 component functions. Judd and co-workers published the S_0, S_1, S_2 functions at 10 nm increments: from these the table accepted by the CIE and reproduced in Table T.2 was calculated using linear interpolation. (Also Table 1 of the CIE standard (CIE, 1998) was calculated using linear interpolation.)

Using the equations in Section 3.1 without following carefully the details of use may lead to slightly different results if the original 10 nm tables, or the 5 nm tables as reproduced in *Section 11. Tables*, or the 1 nm tables of the standard are used. To overcome this problem – and to harmonise colorimetric practice – an alternative method is recommended for evaluation. Since the interpolation of all other colorimetric functions from their original 10 nm values has been done by non-linear interpolation, these new tables have been derived by a Lagrange interpolation method[XX]. The table containing the Lagrange interpolated $S_{0\mathrm{Lag}}$, $S_{1\mathrm{Lag}}$ and $S_{2\mathrm{Lag}}$ values of the S_0, S_1, S_2 functions at 1 nm increments is included on the accompanying CD-ROM. The original Judd and co-workers' 10 nm data were used as input data. Table C. 1 shows the 5 nm sampled set of values.

The starting point for Daylight Illuminant calculation is the set of Equ.'s 3.2, 3.3 and 3.4 of the main publication.

- First one has to calculate the x_D chromaticity coordinate of the daylight phase required using Equ. 3.3 for daylight illuminant correlated colour temperatures from approximately 4000 K to 7000 K, or Equ. 3.4 for daylight illuminant correlated colour temperatures from larger than 7000 K to approximately 25 000 K.
- Then y_D is calculated using Equ. 3.2.
- These Equ.'s are regarded as defining the daylight chromaticity.
- The next step is to calculate the Daylight Illuminant relative spectral power distribution using these x_D, y_D values and Equ. 3.5:[XXI]

$$S(\lambda) = S_0(\lambda) + M_1S_1(\lambda) + M_2S_2(\lambda) \qquad (3.5)$$

The official CIE method uses the $S_0(\lambda)$, $S_1(\lambda)$, $S_2(\lambda)$ functions as defined in Table T.2. These component spectra were originally determined at the full 10 nm increments, and later linearly interpolated to every 5 nm.

To calculate $S(\lambda)$ one has to know the M_1, M_2 factors. The original Judd publication defines these as functions of x_D, y_D as reproduced in Equ. 3.6:[XXI]

$$M_1 = \frac{-1{,}3515 - 1{,}7703x_D + 5{,}9114y_D}{0{,}0241 + 0{,}2562x_D - 0{,}7341y_D}$$

$$M_2 = \frac{0{,}0300 - 31{,}4424x_D + 30{,}0717y_D}{0{,}0241 + 0{,}2562x_D - 0{,}7341y_D} \qquad (3.6)$$

[XX] To be able to determine S_0, S_1, S_2 values at any desired wavelength, the accompanying CD-ROM contains a Lagrange interpolation program.

[XXI] Equ.'s 3.5 and 3.6 are taken from Section 3 of the present document.

The constants in these Equ.'s depend, however, on the sampling of the S functions, see (Kránicz and Schanda, 2000). The main publication overcomes this difficulty by stating that calculations have to be done at 10 nm increments with subsequent linear interpolation.

The M_1, M_2 functions can be written in a general form:

$$M_n = \frac{g_n x_D + h_n y_D + i_n}{j x_D + k y_D + l}$$

where $n = 1$ for Equ. M_1 and $n = 2$ for Equ. M_2.

g, h, i, j, k and l are obtained as the tristimulus values of the $S_0(\lambda)$, $S_1(\lambda)$, $S_2(\lambda)$ functions, and recommended values are listed in Table C 2. Thus e.g.

$$g_1 = 1000 \frac{B_0 D_2 - B_2 D_0}{D_0^2}$$

where e.g.

$$B_0 = \sum_{\lambda_{\min 1}}^{\lambda_{\max}} S_0(\lambda)\, \bar{y}(\lambda) \Delta\lambda \quad \text{and} \quad B_2 = \sum_{\lambda_{\min 1}}^{\lambda_{\max}} S_2(\lambda)\, \bar{y}(\lambda) \Delta\lambda$$

and the D symbols refer to the sum of the three tristimulus values of the form as shown by the B equations for the Y tristimulus values of the S_0, S_1 and S_2 functions.

The tristimulus values will depend on the method of summation, and thus also the g, h, i, j, k and l coefficients will depend on them. This ultimately will have an influence on the x_S, y_S chromaticity coordinates of the daylight phases calculated from the $S(\lambda)$ spectral power distribution according to Equ. 3.5. If these x_S, y_S values deviate from the x_D, y_D values used as the input values in the calculation, a user may feel unsure as to whether he or she has followed the procedure correctly. To avoid such confusion, values of g_n, h_n, i_n, j, k and l have to be calculated for the summation used for calculating $S(\lambda)$. Table C.2 shows the recommended values for the CIE 1931 and CIE 1964 standard observers using the 1 nm standard tables and the Lagrange interpolated $S_{0\text{Lag}}$, $S_{1\text{Lag}}$, $S_{2\text{Lag}}$ tables, both for 1 nm and 5 nm abridged and truncated colour matching tables. Using these g_1 … l coefficients the x_S, y_S values agree with the x_D, y_D values to five decimal places.

Table C.1. $S_{0\mathrm{Lag}}$, $S_{1\mathrm{Lag}}$ and $S_{2\mathrm{Lag}}$ 5 nm sampled spectra.

λ, nm	$S_{0\mathrm{Lag}}$	$S_{1\mathrm{Lag}}$	$S_{2\mathrm{Lsg}}$
300	0,04000	0,02000	0,00000
305	-0,15625	-0,15000	1,15625
310	6,00000	4,50000	2,00000
315	16,56625	12,50500	2,84375
320	29,60000	22,40000	4,00000
325	43,80000	33,40625	6,41875
330	55,30000	42,00000	8,50000
335	57,62500	42,46250	8,50000
340	57,30000	40,60000	7,80000
345	59,69375	41,23750	7,29375
350	61,80000	41,60000	6,70000
355	61,47500	39,58750	5,88125
360	61,50000	38,00000	5,30000
365	65,46875	40,21875	5,80625
370	68,80000	42,40000	6,10000
375	66,40625	40,94375	4,71250
380	63,40000	38,50000	3,00000
385	62,45000	35,98125	2,05000
390	65,80000	35,00000	1,20000
395	79,82500	38,80000	-0,10000
400	94,80000	43,40000	-1,10000
405	101,54375	45,52500	-0,93125
410	104,80000	46,30000	-0,50000
415	106,54375	45,70625	-0,53125
420	105,90000	43,90000	-0,70000
425	100,35000	40,37500	-0,87500
430	96,80000	37,10000	-1,20000
435	104,05000	36,52500	-1,91250
440	113,90000	36,70000	-2,60000
445	120,82500	36,48125	-2,84375
450	125,60000	35,90000	-2,90000
455	126,54375	34,49375	-2,88125
460	125,50000	32,60000	-2,80000
465	123,39375	30,26875	-2,69375
470	121,30000	27,90000	-2,60000
475	121,52500	26,06875	-2,63750
480	121,30000	24,30000	-2,60000
485	117,42500	22,21875	-2,21875
490	113,50000	20,10000	-1,80000
495	112,95625	18,07500	-1,61250
500	113,10000	16,20000	-1,50000
505	112,19375	14,74375	-1,38750
510	110,80000	13,20000	-1,30000
515	108,36250	10,86875	-1,25000
520	106,50000	8,60000	-1,20000
525	107,60000	7,18125	-1,12500
530	108,80000	6,10000	-1,00000
535	107,25000	5,13750	-0,75000
540	105,30000	4,20000	-0,50000
545	104,90625	3,05000	-0,38750
550	104,40000	1,90000	-0,30000
555	102,39375	0,90625	-0,15000
560	100,00000	0,00000	0,00000
565	97,78125	-0,80000	0,10000
570	96,00000	-1,60000	0,20000
575	95,67500	-2,65000	0,26250
580	95,10000	-3,50000	0,50000
585	91,95625	-3,47500	1,25000
590	89,10000	-3,50000	2,10000
595	89,43750	-4,56250	2,69375
600	90,50000	-5,80000	3,20000
605	90,60625	-6,55625	3,68125
610	90,30000	-7,20000	4,10000
615	89,61250	-7,93125	4,43125
620	88,40000	-8,60000	4,70000
625	86,01250	-9,05000	4,83750
630	84,00000	-9,50000	5,10000
635	84,47500	-10,26875	5,88750
640	85,10000	-10,90000	6,70000
645	83,52500	-10,80625	7,01875
650	81,90000	-10,70000	7,30000
655	81,90625	-11,21250	7,91250
660	82,60000	-12,00000	8,60000
665	84,01875	-13,10625	9,25625
670	84,90000	-14,00000	9,80000
675	83,83125	-14,02500	10,19375

λ, nm	$S_{0\text{Lag}}$	$S_{1\text{Lag}}$	$S_{2\text{Lsg}}$
680	81,30000	-13,60000	10,20000
685	76,22500	-12,69375	9,19375
690	71,90000	-12,00000	8,30000
695	72,38125	-12,57500	8,90000
700	74,30000	-13,30000	9,60000
705	76,31875	-13,32500	9,22500
710	76,40000	-12,90000	8,50000
715	69,45625	-11,66250	7,64375
720	63,30000	-10,60000	7,00000
725	66,35000	-10,91875	7,18125
730	71,70000	-11,60000	7,60000
735	75,61250	-12,08750	7,91875
740	77,00000	-12,20000	8,00000
745	72,52500	-11,38750	7,46875
750	65,20000	-10,20000	6,70000
755	54,40625	-8,66250	5,73125
760	47,70000	-7,80000	5,20000
765	57,28125	-9,40000	6,24375
770	68,60000	-11,20000	7,40000
775	68,04375	-11,00000	7,22500
780	65,00000	-10,40000	6,80000
785	65,58750	-10,50625	6,90000
790	66,00000	-10,60000	7,00000
795	64,04375	-10,25000	6,76875
800	61,00000	-9,70000	6,40000
805	56,48750	-8,88125	5,87500
810	53,30000	-8,30000	5,50000
815	55,43125	-8,68125	5,71875
820	58,90000	-9,30000	6,10000
825	61,71875	-9,79375	6,43125
830	61,90000	-9,80000	6,50000

Table C.2. g_1 to l coefficients for CIE 1931 and 1964 standard observers and 1 nm standard tables using the **Lagrange** interpolated S_0, S_1 and S_2 functions both for 1 nm and 5 nm sampling.

Coeff.s	CIE 1931 St. Obs. 1 nm sampling	CIE 1964 St. Obs. 1 nm sampling	CIE 1931 St. Obs. 5 nm sampling	CIE 1964 St. Obs. 5 nm sampling
g_1	-1,77864	-1,57049	-1,77861	-1,57049
h_1	5,90745	5,56450	5,90757	5,56460
i_1	-1,34666	-1,31211	-1,34674	-1,31215
g_2	-31,44505	-30,15166	-31,44464	-30,15139
h_2	30,06408	31,07906	30,06400	31,07931
i_2	0,03656	-0,73912	0,03638	-0,73928
j	0,25540	0,21249	0,25539	0,21250
k	-0,73218	-0,71591	-0,73217	-0,71592
l	0,02387	0,04663	0,02387	0,04663

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APPENDIX D. REVERSE TRANSFORMATION FROM VALUES *L**, *a**, *b** TO TRISTIMULUS VALUES *X, Y, Z*

In Information Technology (IT) transformations are necessary from the tristimulus values X, Y, Z to the values L^*, a^*, b^* and in the reverse [R] direction. Therefore it was decided at the TC meetings in Veszprém in August 2002 to include a proposed reverse transformation into CIE-Publication 15 "Colorimetry". The first transformations are given in the main part of the document (see Section 8.2.1) and the reverse transformations are in this Appendix D.

Reverse transformation:

Calculate from the values L^*, a^*, b^*

$$f(Y/Y_n) = (L^* + 16)\,/\,116 \tag{D.1}$$

$$f(X/X_n) = a^*/\,500 + f(Y/Y_n) \tag{D.2}$$

$$f(Z/Z_n) = f(Y/Y_n) - b^*/200 \tag{D.3}$$

Calculate then the tristimulus values X, Y, Z from:

$$X = X_n[f(X/X_n)]^3 \qquad \text{if} \quad f(X/X_n) > 24/116 \tag{D.4}$$

$$X = X_n[f(X/X_n) - 16/116] \text{x} (108/841) \qquad \text{if} \quad f(X/X_n) \leq 24/116 \tag{D.5}$$

$$Y = Y_n[f(Y/Y_n)]^3 \qquad \text{if} \quad f(Y/Y_n) > 24/116 \text{ or } L^* > 8 \tag{D.6}$$

$$Y = Y_n[f(Y/Y_n) - 16/116] \text{x} (108/841) \qquad \text{if} \quad f(Y/Y_n) \leq 24/116 \text{ or } L^* \leq 8 \tag{D.7}$$

$$Z = Z_n[f(Z/Z_n)]^3 \qquad \text{if} \quad f(Z/Z_n) > 24/116 \tag{D.8}$$

$$Z = Z_n[f(Z/Z_n) - 16/116] \text{x} (108/841) \qquad \text{if} \quad f(Z/Z_n) \leq 24/116 \tag{D.9}$$

Note: Integer ratios, e. g. 16/116, 841/108, 108/841, are used for both transformations to avoid rounding errors. The break point between the non-linear and the linear part of the values L^* is at $L^* = 8$ as proposed by Pauli (1976).

Reference

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APPENDIX E. INFORMATION ON THE USE OF PLANCK'S EQUATION FOR STANDARD AIR

According to the Planck's law, the spectral radiance of a blackbody at thermodynamic temperature T [K] in a medium having index of refraction n is given by

$$L_{e,\lambda}(\lambda,T)=\frac{c_1 n^{-2}\lambda^{-5}}{\pi}\left[\exp\left(\frac{c_2}{n\lambda T}\right)-1\right]^{-1} \tag{E.1}$$

where $c_1 = 2\pi hc^2$, $c_2 = hc/k$, h is Planck's constant, c is the speed of light in vacuum, k is the Boltzmann constant, and λ is the wavelength in the medium. Since T is measured on the International Temperature Scale, the value of c_2 used in colorimetry should follow that adopted in the current International Temperature Scale (ITS-90) (Preston-Thomas, 1990; Mielenz et al., 1991), namely $c_2 = 1{,}4388 \times 10^{-2}$ m K. The official value of c_1 is provided by the Committee on Data for Science and Technology (CODATA), and is $c_1 = 3{,}741\,771 \times 10^{-16}$ W m^2 (Mohr and Taylor, 2000). The value of c_1, however, is not relevant in colorimetry where only the relative spectral distribution of Planck's radiation is used.

The value of n of air depends on the partial pressure of each constituent of the air and is also wavelength dependent, but for standard air (dry air at 15°C and 101 325 Pa, containing 0,03 percent by volume of carbon dioxide), it is approximately 1,00028 throughout the visible region. This value has been used in photometry and radiometry for a number of years (Blevin, 1972), and is confirmed from the latest physical data showing n more precisely as a function of wavelength (Cohn et al., 2003).

Using Equ. E.1 with n = 1,00028, CIE standard illuminant A can be calculated, with negligible differences from the published values (1 count or less in the 6th significant digit), by

$$S_A(\lambda)=\frac{L_{e,\lambda}(\lambda,\ T)}{L_{e,\lambda}(560\,\text{nm},\ T)} \tag{E.2}$$

where $c_2 = 1{,}4388 \times 10^{-2}$ m·K (ITS-90) and T = 2854,742 K. This temperature T is the value assigned to CIE standard illuminant A in the Planck's equation for standard air. This implies that correlated colour temperature of CIE standard illuminant A should be ~2855 K, a change of ~1 K from the previous value (2856 K). However, TC 1-48 did not come to a consensus at this time to revise the definition of correlated colour temperature using n = 1,000 28, because the form of Planck's equation in vacuum (n is equal to exactly 1) has been used historically and it was concerned that such a revision would cause a small discontinuity from the values obtained in the past (though the difference is practically insignificant and matters only in rigorous calculations). It was also concerned that the revision still would not describe Planck's radiation in air perfectly; the value of n is slightly wavelength dependent, which would bring the CIE standard illuminant A deviating very slightly off from a perfect Planckian radiator (though the deviation would be less than 0,005 % – calculated using data in Cohn et al., 2003.

Therefore, in the current recommendation in CIE 15:2004, colour temperature and correlated colour temperature are calculated using Equ. E.1 with n = 1 (exactly 1), thus no change from the previous practice. This recommendation may be subject to change in the future.

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EXPLANATORY COMMENTS

1 CIE Division 1 (at its meeting in Rochester in 2001) and CIE Division 2 (at its meeting in Gaithersburg in 2001) confirmed that wavelengths measured in standard air (dry air at 15°C and 101 325 Pa, containing 0,03 percent by volume of carbon dioxide) should be used in all CIE publications.

2 This recommendation deviates from earlier recommendations regarding CIE sources A, B, C (CIE, 1931; CIE, 1951). A distinction is made between *illuminant* and *source*. The term source refers to a physical emitter of light, such as a lamp or the sun and sky. The term illuminant refers to a specific spectral power distribution, not necessarily provided directly by a source, and not necessarily realizable as a source. The present recommendation first defines illuminants by relative spectral power distributions and then sources. The definition of the sources is considered secondary, as it is conceivable that new developments in lamps and filters will bring about improved sources that represent the illuminants more accurately and are more suitable for laboratory use. At present, no recommendation has been made for a source representing standard illuminant D65. The original recommendations regarding standard illuminant D65 (originally called D_{65}) and other illuminants D representing daylight of different correlated colour temperatures are given in CIE Proceedings (CIE, 1963; CIE, 1967).

In 1968 the Comité International des Poids et Mesures modified the "International Practical Temperature Scale, 1948 (amended 1960)" and the value of the radiation constant c_2 was set equal to $1{,}4388 \cdot 10^{-2}$ m·K. This modification has affected the colour temperature or correlated colour temperature of the CIE illuminants and sources.

3 Equation 3.1 is equivalent to and can be derived from the expression

$$S(\lambda) = 100\, M_{\mathrm{e},\lambda}(\lambda,T) \,/\, M_{\mathrm{e},\lambda}(\,560,\,T\,), \qquad \text{(EN.1)}$$

where

$$M_{\mathrm{e},\lambda}(\lambda,T\,) = c_1\, \lambda^{-5}\, [\exp(c_2\,/\lambda T) - 1]^{-1}, \qquad \text{(EN.2)}$$

λ is the wavelength (in nanometres), and the ratio $c_2\,/T$ is given by

$$c_2\,/T = 1{,}435 \times 10^7 / \,2\,848 \text{ nm} \qquad \text{(EN.3)}$$

Since the numerical value of c_1 cancels out of Equ. E.1, this definition of CIE standard illuminant A involves no assumptions about the numerical values of c_1, c_2, and T other than the ratio defined in Equ. E.3. (See CIE 1998c).

CIE standard illuminant A was originally defined in 1931 as the relative spectral power distribution of a Planckian radiator of temperature

$$T_{\mathrm{CIE\ 1931}} = 2\,848 \text{ K}, \qquad \text{(EN.4)}$$

the value of the second radiation constant c_2 then being taken as

$$c_{2,\ \mathrm{CIE\ 1931}} = 1{,}435\,0 \times 10^{-2} \text{ m·K}. \qquad \text{(EN.5)}$$

The form of the definition as now printed was carefully chosen to ensure that CIE standard illuminant A was defined as a relative spectral power distribution and not as a function of temperature; as explained above, the definition of the relative spectral power distribution has not changed since 1931 and Equ. 3.1 simply expresses it in a general form.

What has changed is the temperature assigned to this distribution. The value of c_2 given in Equ. E.5 and used by the CIE in 1931 is different from the respective values, $c_{2,\mathrm{ITS\text{-}27}} = 14\,350$ µm·K, $c_{2,\mathrm{IPTS\text{-}48}} = 14\,380$ µm·K, and $c_{2,\mathrm{IPTS\text{-}68}} = c_{2,\mathrm{ITS\text{-}90}} = 14\,388$ µm·K, that were assigned to this constant in the International Temperature Scales of 1927, 1948, 1968 and 1990, respectively. Although this has had no effect on the relative spectral power distribution of CIE standard illuminant A, the correlated colour temperatures of sources recommended for laboratory realizations have been different, over the years, depending on the values of c_2 used. As may be seen from Equ. E.3, the colour temperatures associated with CIE standard illuminant A on the various international temperature scales referred to above were T_{27} = 2 848 K, T_{48} = 2 854 K, and $T_{68} = T_{90}$ = 2 856 K, respectively.

Another change over the years has been the explicit decision that the wavelengths in Equ.'s 3.1, E.1 and E.2 should be taken as being in standard air despite the fact that the equations are derived from the form of Planck's equation in vacuum. The Planck's equation requires a refractive index term if the wavelengths refer to a medium other than vacuum. Illuminant A was originally defined in 1931 as the relative spectral distribution of a Planckian radiator but, historically, wavelengths in standard air have always been used in photometry and colorimetry. Despite this, CIE/ISO 10526:1991 and its revision, ISO 10526:1999(E)/CIE S 005-1998, stated that the term λ denoted a wavelength in vacuum. This was inconsistent with previous CIE publications, but the situation was not addressed until recently. CIE Division 1 and Division 2 agreed in 2001 that all wavelengths used in CIE Publications should be in standard air (explanatory comment 1). The current document follows this decision. ISO 10526:1999/CIE S 005-1998 is also to be revised to reflect this change (CIE S 014-2 to be published). The use of standard air wavelengths means that the colour temperature of illuminant A is very slightly different from the values of T used in the above equations but the difference is insignificant for all practical purposes. See Appendix E for further information on the use of Planck's equation for standard air.

It is important to note that the numerical values defined for CIE standard illuminant A have never changed. The only issues have been the change of temperature scale and the use of standard air. The 1 nm tables given in CIE standard S005 and reproduced on the CD-ROM accompanying this report are consistent with Equ. 3.1. In cases where calculation at 5 nm intervals is found to produce no significant error, the data in Table T.1 can be used.

The rounded values of $S(\lambda)$ for CIE standard illuminant A given in Table T.1 show, in several instances, small and insignificant discrepancies of one unit in the last decimal from corresponding values commonly used in various publications. The values given in Table T.1 are the correctly rounded values and agree with those of the standard.

4 The correlated colour temperatures are affected by the numerical value of the radiation constant c_2. In accordance with the *International Practical Temperature Scale, 1948, amended 1960* which was in use at the time when the procedure for calculating daylight illuminants was adopted by the CIE, the value of c_2 was equal to 1,438 0 x 10^{-2} m·K. With this value, the correlated colour temperature of illuminant D65 is approximately equal to 6500 K. The change of c_2 to the value of 1,438 8 x 10^{-2} m·K (International Practical Temperature Scale, 1968) increases the correlated colour temperatures of illuminant D65 by the factor 1,4388/1,4380. Thus the correlated colour temperature increases by approximately 4 K.

5 The method required to calculate the values for the relative spectral power distributions of illuminants D50, D55, D65, and D75, in Table T.1 is as follows

1. Multiply the nominal correlated colour temperature (5000 K, 5500 K, 6500 K or 7500 K) by 1,4388/1,4380.
2. Calculate x_D and y_D using the equations given in the text.
3. Calculate M_1 and M_2 using the equations given in the text.
4. Round M_1 and M_2 to three decimal places.
5. Calculate $S(\lambda)$ every 10 nm by $S(\lambda) = S_0(\lambda) + M_1 \cdot S_1(\lambda) + M_2 \cdot S_2(\lambda)$ using values of $S_0(\lambda)$, $S_1(\lambda)$ and $S_2(\lambda)$ from Table T.2.
6. Interpolate the 10 nm values of $S(\lambda)$ linearly to obtain values at intermediate wavelengths.

This method should also be followed to calculate the relative spectral power distribution of illuminants at other nominal correlated colour temperatures.

6 This recommendation supersedes the original (CIE, 1931) and is in accordance with a later agreement (CIE, 1959).

7 This recommendation has been changed from the original (CIE, 1931) method several times, see joint meeting of E-1.3.1, E-1.3.2 and E-1.2 on June 28, 1967 in Washington (CIE, 1967). The present version is the result of a major updating, as colorimetric practice has shown that more precise definitions of measuring geometry are needed. Regarding practical measurements see also (CIE, 1998d).

8 Where single-beam integrating spheres are used, a correction for the reduction of sphere efficiency caused by sample absorption is necessary. Without such a correction, the instrument will give a non-linear output. The corrected reflectance is given by:

$$\rho_S = R(\lambda)\frac{1-\rho_w(\lambda)\cdot(1-\Sigma_i f_i)}{1-\rho_w(\lambda)\cdot(1-\Sigma_i f_i)-f_S\cdot(\rho_r(\lambda)-R(\lambda))},$$

where $R(\lambda)$ is the uncorrected reflectance of the sample referred to the perfect reflecting diffuser, $\rho_w(\lambda)$ is the diffuse/diffuse spectral reflectance of the sphere wall, f_i is the fractional area of the i th port, f_S is the fractional area of the sample port, and $\rho_r(\lambda)$ is the reflectance of the reference standard. The above equation assumes ideal integrating characteristics for the sphere and that the other ports have zero effective reflectance.

9 These recommendations are based on the originals (CIE, 1931; CIE, 1959; CIE, 1963).

The recommendations given in this document regarding the CIE 1931 standard colorimetric observer data deviate from the originals in several ways. The CIE 1931 standard colorimetric observer is now defined by the colour-matching functions $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, $\bar{z}(\lambda)$ from 360 nm to 830 nm at 1 nm intervals (see Table 1 in CIE, 1986a). From these data the abridged data reproduced in Table T.4. have been derived by selecting every fifth value and rounding to six decimal figures.

10 Colour-matching functions $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, $\bar{z}(\lambda)$ are relative tristimulus values of monochromatic radiators of equal radiant power related to a set of reference stimuli [X], [Y], [Z].

11 The colour-matching functions $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, $\bar{z}(\lambda)$ given in Table T.4 agree closely with those defined originally in 1931. Three minor changes have been introduced: at λ =775 nm the new values of $\bar{x}(\lambda)$ is 0,000 059 instead of 0,0000; at λ = 555 nm $\bar{y}(\lambda)$ is 1,0000 instead of 1,0002; and at λ = 740 nm $\bar{y}(\lambda)$ is 0,000 249 instead of 0,0003. These changes are considered insignificant in most colorimetric computations. From these corrected tables the CIE standard colorimetric observer (CIE, 1986a) was determined.

12 For daylight illuminant D65 2,464 scotopic trolands corresponds to 1 photopic troland (see Wyszecki & Stiles, 1982, p. 104). Rod saturation in 9° extrafoveal vision occurs at about 2000-5000 scotopic trolands (see Wyszecki & Stiles, 1982, p. 547). Thus rod saturation would occur at a photopic light level of between 812-2 029 trolands. Working from the Table in Le Grand (1968, p. 106) that takes into account variation of pupil size with light level and the Stiles-Crawford effect, this would correspond to 130-380 cd/m^2 (kind contribution by J. Pokorny).

13 These recommendations formalise procedures for practical colorimetric calculations.

14 CIE 15:2004 states only the facts on abridgement and truncation. A forthcoming publication of a tutorial nature will detail the procedures, with examples showing good practice.

15 This recommendation is based on the original (CIE, 1955) amended in CIE Bulletin No. 3, (CIE, 1957). The use of boldface Roman letters as symbols for vector notations is another alternative which was added in the first edition of this publication. In the present 3rd edition of the document the recommendation to use "Gothic letters" has been dropped.

16 These recommendations are based on those given in supplement No. 2 to the 1st edition of this publication (CIE 1978), the publication on "Industrial colour difference evaluation" (CIE 1995a) and the publication "Improvement to industrial colour difference evaluation" (CIE, 2001a), with some amendments agreed at TC 1-48 and TC 1-59 meetings in Veszprém, 2002 and San Diego, 2003.

17 In June 1967 the CIE colorimetry committee recommended to the National Committees of the CIE a detailed working program for pursuing the problem of colour difference calculations (Wyszecki, 1968). Further guidelines were published in 1978 (Robertson, 1978). The parametric effects in colour difference evaluation were summarised in 1993 (CIE, 1993), and an advanced colour difference calculation method was published in 1995 (CIE, 1995a). Recent findings are summarized in (CIE, 1999a). The currently recommended colour difference formula is described in (CIE, 2001a).

18 Equations (8.7), (8.9) and (8.11) are based on a suggestion by Pauli (1976), according to which below $L^*=8$ a linear $L^*=f(Y)$ dependence should be followed. In CIE 15.2 decimal approximations were used. This brought the break point of $Y/Y_n = \left(\frac{24}{116}\right)^3$ to 0,008856, and the exact value of $\frac{1}{3}\times\left(\frac{116}{24}\right)^2$ to 7,787. At the Technical Committee TC 1-48 meeting in Veszprém, 2002, the committee agreed that to secure the continuity at the break between the two parts of the equations $f(A_i / A_{i,n})$, where $A_1= X$, $A_2= Y$, $A_3= Z$, $\left(\frac{24}{116}\right)^3$ should be written instead of its approximate value of 0,008 856, and similarly $\frac{1}{3}\times\left(\frac{116}{24}\right)^2=\frac{841}{108}$ should be used instead of 7,787 in Equ.'s (8.7), (8.9) and (8.11).

19 This recommendation is essentially the same as the original (CIE, 1948).

20 Previous two publications *Special metamerism index: Change in illuminant* (Supplement No. 1 to 1st edition of CIE 15-1971 (CIE, 1971) and *Special metamerism index: Change in observer* (CIE, 1989) have been combined in this version of CIE 15. CIE 80-1989 is, however, still current, and provides more details on the subject.

21 Table T.6 gives the relative spectral power distributions of 15 FL-illuminants that represent 15 different types of fluorescent lamps.

The FL1 to FL12 group of lamps were also included in 15.2 (there as F1 to F12). The further spectra are enumerated by starting with the number 3, reflecting that they have been first introduced when elaborating version 3 of CIE 15. Each of the relative spectral power distributions in the standard halophosphate ("St-Halo") group (FL3.1-3.3) consists of two semi-broadband emissions of antimony and manganese activations in a calcium halo-phosphate phosphor. The "DeLuxe" group (FL 3.4-3.6) are more or less enhanced in colour rendering properties as compared to the "St-Halo" group, usually using multiple phosphors. Relative spectral power distributions of the "DeLuxe" group are flatter and have a wider range in the visible spectrum than the "St-Halo" group. Relative spectral power distributions of the "Three-band" group (FL3.7-3.11) consist mostly of three narrow-band emissions in the red, green, and blue wavelength regions. In most cases the narrow-band emissions are caused by ternary compositions of rare-earth phosphors. FL3.12-3.14 represent modern multiband fluorescent lamp spectra. FL3.15 is the spectrum of a fluorescent lamp developed commercially to simulate D65, taken from JIS 8716 (JIS, 1991).

22 In many applications high-pressure metal-halide and sodium lamps are gaining in importance. Spectra, as reproduced in Table T.7, with colorimetric data shown in Table T.9, are representative spectra of these classes of sources and could be used for checking colorimetric properties of objects when illuminated with such lamps.

23 This is a new Section in the document based on CIE 51, first published in 1981 for assessing illuminants D55, D65 and D75 (CIE, 1981), and amended in 1999 with a technique to assess the quality of D50 illuminants (CIE, 1999b).

24 The daylight simulators having suitable categories, as assessed by the method in this report, can be used to simulate standard illuminants D50, D55, D65 and D75 for visual matching of object-colour samples, and for reproducing the spectral total radiance factor of samples for instrumental colorimetry and spectrophotometry; the samples may be non-fluorescent or fluorescent.

The daylight simulators may also be used for the visual appraisal of the colour rendering properties of other light sources, though the present method of evaluation for daylight simulators does not directly relate to the evaluation of colour rendering properties.

25 This recommendation was published for the first time in the edition 15.2-1986 of this publication.

26 This section has been updated according to the decisions of CIE Division 1 obtained at its meeting in Rochester, USA, 2001.